

OPERATIONAL DIAGNOSTIC APPLICATIONS OF ISENTROPIC ANALYSIS

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ABSTRACT

Basic isentropic analysis techniques are discussed and comparisons with pressure surface analyses presented to illustrate several advantages of mapping synoptic and sub-synoptic scale flow patterns in the isentropic framework. Since detailed isentropic maps are not presently available to operational meteorologists, a method for generating simple isentropic charts on an operational basis, utilizing the standard 850mb, 700 mb, and 500 mb pressure maps, is presented. Isentropic maps constructed with the operational technique are then related to conventional surface and upper air analyses to demonstrate the diagnostic capability of this approach.

1. Introduction

A renewed interest in using isentropic analysis for the study of synoptic scale weather disturbances is slowly developing, due largely to the work of Danielsen (1959, 1961, 1964, 1966), Reiter (1963, 1972), Bleck (1973, 1974), Shapiro (1975; and Hastings 1973), and Johnson (1970; and Downey, 1975A, 1975B, 1976) among others. However, the reemergence of isentropic coordinates as a valuable diagnostic tool has been restricted to a relatively small group of research meteorologists. Operational meteorologists have ignored the isentropic framework since the late forties when pressure was chosen as the vertical coordinate for constructing upper level charts.¹ The continued disuse of isentropic analysis for diagnosing weather features by operational and even research meteorologists is all but insured since, for the most part, they have not been taught how to interpret isentropic maps, are not acquainted with the advantages of them, and do not have any apparent access to them. Thus, the situation of avoiding isentropic analysis while relying solely on isobaric analysis is perpetuated.

There are several important advantages for mapping atmospheric flow fields in the isentropic framework:

- 1) For the synoptic time scale, isentropic surfaces act as material surfaces which are penetrated significantly only by such diabatic processes as the release of latent heat in rising saturated air or the addition of sensible heat in the planetary boundary layer (Rossby, et al., 1937).
- 2) Since an isentropic surface can vary in height (z) and pressure (p), "horizontal" flow along the surface includes the diabatic component of the vertical motions dz/dt and dp/dt which would otherwise be derived as a separate velocity component in either Cartesian or isobaric coordinates. This factor allows for a more accurate determination of atmospheric trajectories on isentropic surfaces than is possible in the other coordinate systems (Danielsen, 1961).
- 3) The tendency to consider only the horizontal advection of moisture along a pressure surface and ignore the vertical transport through the surface is avoided with isentropic maps. The "horizontal" moisture transport along an isentropic surface implicitly includes the vertical transport which would have to be measured separately in pressure space. Therefore, a more coherent three dimensional depiction of moisture transport is attained using isentropic maps rather than the standard 850mb or 700

mb charts on which moisture patterns tend to be discontinuous and fragmentary (Oliver and Oliver, 1951).

The purposes of this paper are to acquaint operational meteorologists with several isentropic analysis techniques which apply to diagnosing synoptic and sub-synoptic scale flow patterns (Saucier, 1955), and to illustrate the advantages of the isentropic framework for better depicting moisture transport and for providing a simple and direct method to estimate synoptic scale vertical motion. Since detailed isentropic maps are not presently available on an operational basis, a method for construction such maps from the standard isobaric charts is presented and examples discussed. These examples demonstrate that the operational technique for generating isentropic maps can be used to diagnose the interactions among the lower, middle and upper tropospheric layers which are responsible for important weather features.

2. Examples of Basic Isentropic Analysis

Examples of isentropic maps² are illustrated in Figures 1A, 1B, and 4. The light solid lines in Fig. 1A are isolines of the Montgomery stream function (ψ). The ψ analysis is analogous to the analysis of geopotential heights on pressure surfaces with the geostrophic wind vector being a function of the gradient of ψ .

The pressure analysis (dashed line, Fig. 1B) is a measure of the slope of an isentropic surface with lower (higher) pressure implying that the surface is at a higher (lower) altitude. The magnitude of the thermal wind vector is a function of the pressure gradient on an isentropic surface. Thus large pressure gradients on an isentropic surface imply the existence of large vertical wind shears and represent baroclinic or frontal zones. The mathematical representation of these important physical relationships can be found in the Appendix A.

The analysis of the pressure difference between two isentropic surfaces (p) provides a measure of stability between isentropic surfaces (in this case 5k apart, Fig. 1B). Where Δp is large the layer is more nearly adiabatic and thus relatively unstable, while small Δp implies a more stable region often related to inversion layers (Fig. 2). The large values of Δp at the 750mb and 650mb levels over Kansas and Oklahoma (Fig. 1B) represents a very unstable region which overlaid a warm, moist air flow from the Gulf region. This condition was partly responsible for a violent tornado outbreak in Kansas and eastern Oklahoma which occurred by 0000GMT 26 May 1973.

¹See Bleck (1973) for an excellent historical review of the demise and rebirth of isentropic coordinates, and the general acceptance of pressure coordinates which occurred in the thirty-year period 1930-1960.

²The maps are generated from a RAOB processing program written by Whittaker, University of Wisconsin, Madison.

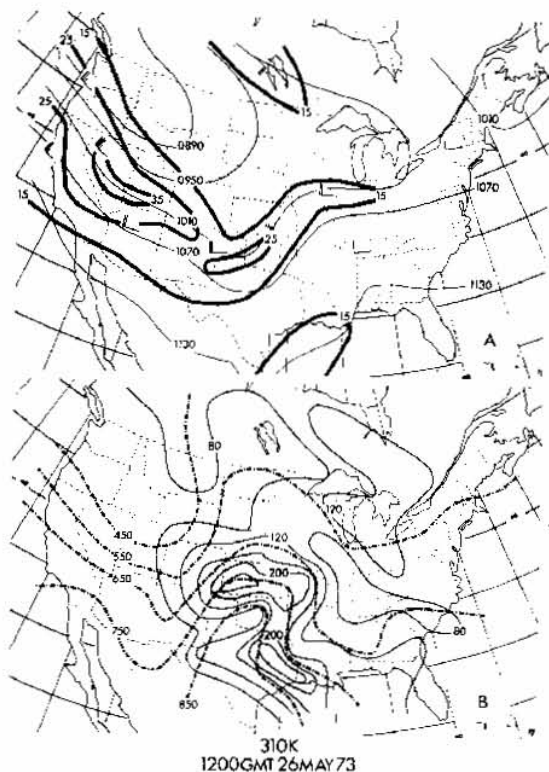


FIG. 1: Isentropic analyses on 310K surface for 1200 GMT 26 May 1973.
A; isotachs (thick solid) in m sec^{-1} ψ analysis (thin solid) $\times 10^4 \text{ m}^2 \text{ sec}^{-2}$
B; p (thin solid) for 310K to 315K layer, in mb. Pressure (dot dashed) for 310K surface, in mb.

2. 1 Moisture analysis

Figures 3 and 4 compare the analysis of mixing ratio on an isentropic surface with mixing ratio analyses on 850 mb and 700 mb charts. On the 850mb map (Fig. 3B) the moisture maximum is located in southern Arkansas-northern Louisiana downwind from relatively smaller values of the mixing ratio and detached from the obvious moisture source, the Gulf of Mexico. We could infer that the water vapor is rising up through the 850mb surface at this location rather than being advected along the surface. At the 700 mb level (fig. 3C) there is an isolated moisture maximum from northeast Texas extending to northern Arkansas. Again with the maximum downwind of smaller values of the mixing ratio, we cannot simply consider the "horizontal" advection of water vapor along the pressure surface but must also account for the vertical transport through the surface. This vertical moisture transport is clearly illustrated on the 310K isentropic surface (Fig. 4A). The continuous flow of moisture from the western Gulf rises from below 850 mb in east Texas up to 800 mb in southwest Arkansas to the 700mb level in northern Arkansas. There the moisture flow splits into two branches; one extending north and northwestward toward Iowa and the Dakotas, the other northeastward into southern Illinois. The vertical transport in Texas, Louisiana and Arkansas region which had to be inferred using pressure maps is clearly represented on this isentropic surface.

2. 2 Estimating vertical motion

The vertical motion field is the most important aspect in diagnosing current weather conditions and forecasting possible changes. At present, however, there is no NMC product that provides a quantitative vertical motion estimate based upon *current* RAOB data. This has forced

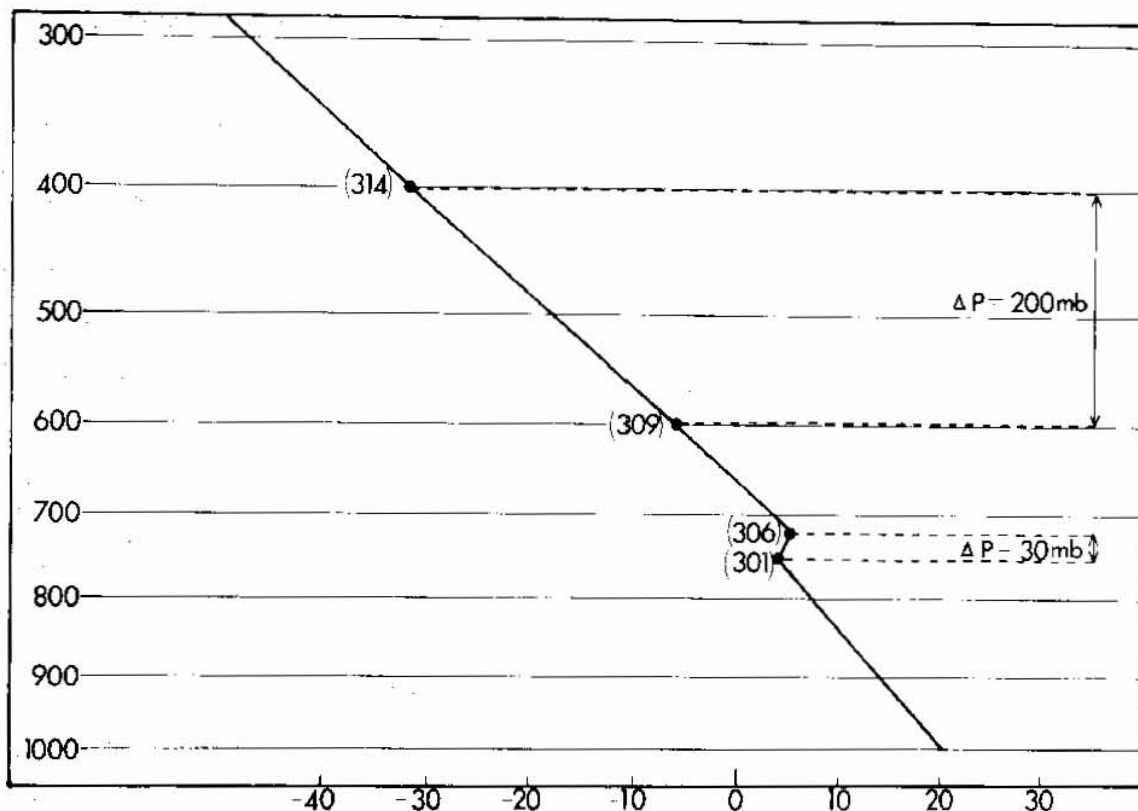


FIG. 2: Hypothetical temperature sounding (potential temperature in parentheses) with p for selected 5K increments indicated on right.

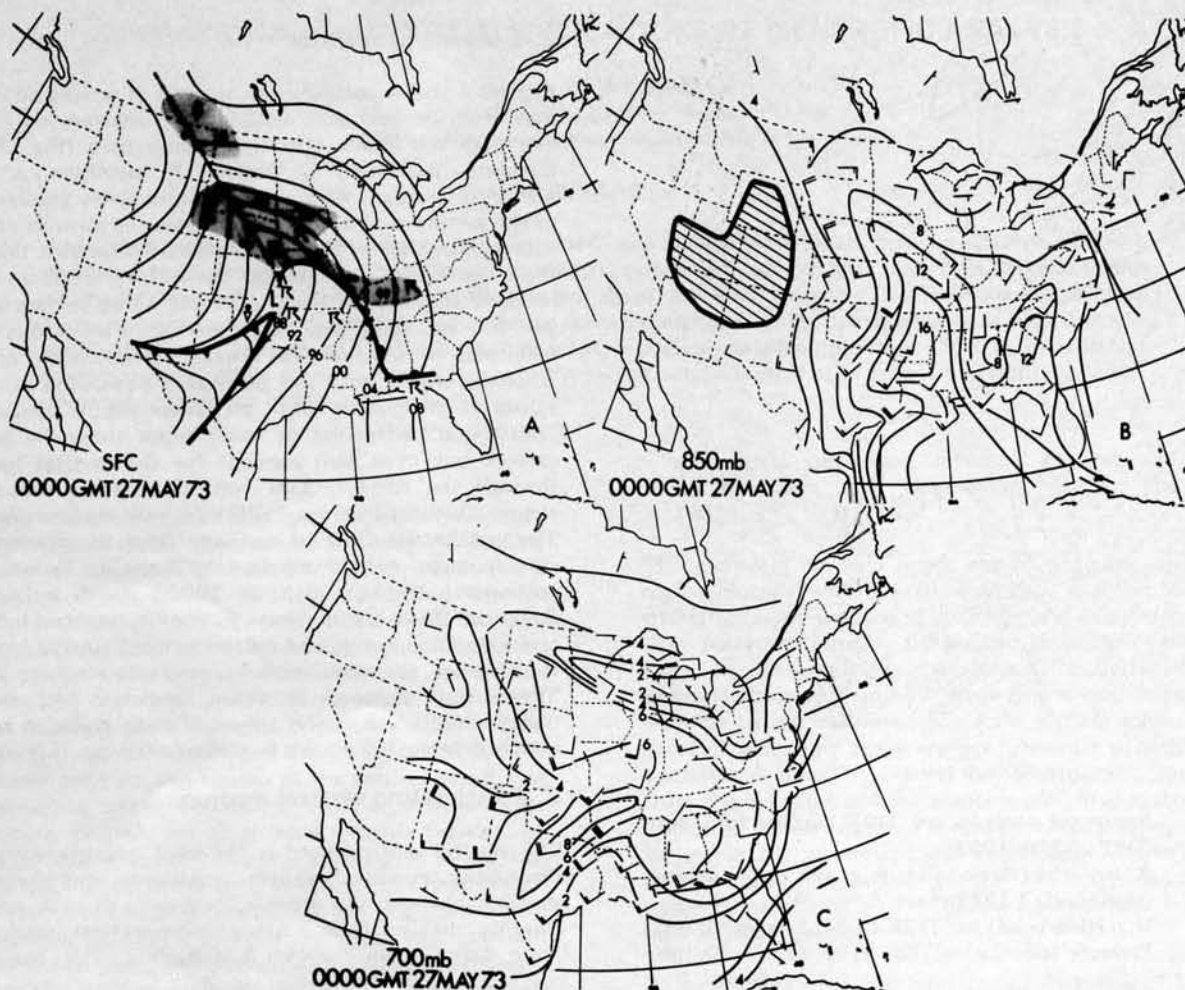


FIG. 3: A: surface pressure and precipitation analyses B: 850mb mixing ratio (g/kg) analysis C: 700mb mixing ratio (g/kg) analysis for 0000 GMT 27 May 1973.

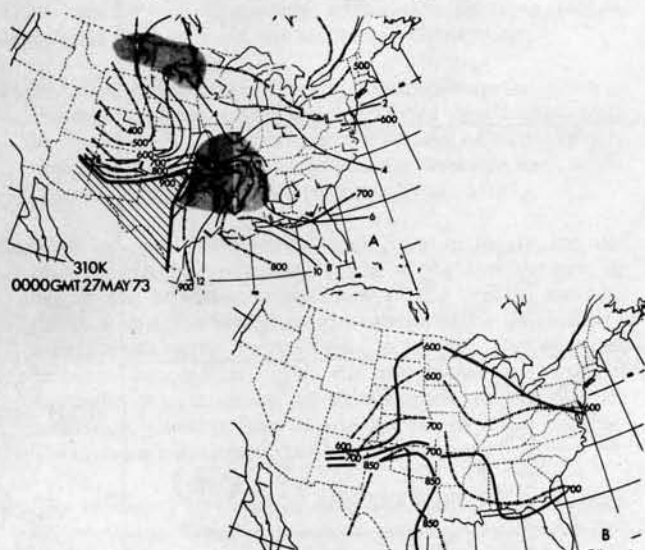


FIG. 4: A; 310K isentropic surface for 0000 GMT 27 May 1973 with pressure in mb (dot dashed line) and mixing ratio (thin solid). Hatched area, surface is underground. Wind barbs in m sec^{-1} . Shaded area is region of upward vertical velocity as estimated from advective term (see text). B; pressure analysis for 1200 GMT 26 May 1973 (dot dashed) and 0000 GMT May 1973 (thick solid).

meteorologists to rely on *indirect* means, i.e., temperature and vorticity advection, to qualitatively depict regions of vertical motion. Quantitative estimates of the vertical velocity using this approach require a solution for the " ω " equation which accounts for these two physical processes but is very difficult to obtain.

Analyses of wind and pressure on an isentropic surface can yield a *direct* quantitative estimate of the vertical velocity (ω) simple from the expansion of the total derivative dp/dt where,

$$\omega \equiv \frac{dp}{dt} = \frac{\partial p}{\partial t} + \vec{v} \cdot \nabla_{\theta} p + \frac{d\theta}{dt} \frac{\partial p}{\partial \theta} \quad (1)$$

$$[1] \quad [2] \quad [3]$$

The local pressure change (term 1) over a 12 hour period can be estimated at specific locations using successive maps. The horizontal advective change (term 2) is determined directly from the current map. Air flowing from high to low pressure constitutes rising motion ($-\omega$). The diabatic heating term (term 3) is important in regions of clouds and precipitation where latent heat is released ($d\theta/dt > 0$) enhancing upward vertical motion ($\partial p/\partial \theta$ is always assumed negative). The third term is difficult to measure given the uncertainty in calculating $d\theta/dt$.

In the example presented in Figure 4A the advective change (term 2) is quite significant in northeastern Texas, Arkansas, southern Missouri and southern Illinois (shaded region in Fig. 4A), and we could expect this to represent upward vertical motion if the local change (term 1) remains small positive or is in fact negative. By comparing the pressure

analyses between the current and previous maps (Fig. 4B), we find the local change ($\partial p / \partial t$) in this region to be small compared to the advective change. In central Arkansas $\partial p / \partial t \approx 2.0 \times 10^{-3}$ mb sec⁻¹ compares to $V \cdot \nabla \theta p \approx -8 \times 10^{-3}$ mb sec⁻¹ compares to $V \cdot \nabla \theta p \approx -12.0 \times 10^{-3}$ mb sec⁻¹. In general, where wind speeds are large and directed along the pressure gradient the advective change (term 2) will dominate the other terms (Saucier, 1955) and therefore yield a reasonable estimate of significant synoptic scale vertical motion. The addition of latent and sensible heat, which is difficult to account for directly, will further increase the upward vertical motion associated with the advective change and is probably significant in eastern Oklahoma and Arkansas in this case. Note that the area of rising motion (Fig. 4A) lies to the south and extends into the regions of continuous precipitation in the central plains, while the rising motion in the western Dakotas coincides with the separate area of precipitation in the northern plains (fig. 3A). In northern Arkansas and southern Missouri, where a significant amount of water vapor is being transported up the isentropic surface, a separate outbreak of severe thunderstorms developed by 0400 GMT 27 May, ahead of the line of thunderstorms indicated on the 0000 GMT map in northeastern Oklahoma. The sinking air in northern Texas, western Oklahoma and Kansas accounts for the bulge of drier air moving into that region.

3. Construction of Isentropic Charts from Standard Pressure Maps

Detailed isentropic maps are not available to operational meteorologists at the present time. However, isentropic maps can be constructed utilizing the temperature, dewpoint and wind information on the 850mb, 700 mb and 500 mb pressure charts. The unique relationship between θ , T , and P specified by Poisson's equation implies that an isotherm on a pressure surface converts to an isobar on an isentropic surface and provides the means for constructing simple isentropic maps.

To construct the isentropic chart, first isolate an area of interest on the 850mb, surface, e.g. where a low level jet coexists with large dewpoint values. Choose an isotherm which runs through that area, not necessarily using the objectively analyzed isotherms, but rather analyzing for an isotherm which passes through the wind maximum and through or near as many station locations as possible.

Since the chosen isotherm at the 850mb level corresponds to a unique isentrope (θ_k), which we determine from a thermodynamic diagram, the isotherm can be transformed to an 850 mb isobar on an isentropic surface. For example, the 4C isotherm on the 850 mb map is equivalent to the 850mb isobar on the 291K isentropic surface (Fig. 5A). Next an interpolation of the wind velocity and mixing ratio³ to the isotherm on the 850mb surface yields the same information to the 850mb isobar on the θ_k isentropic map. The interpolation is a result of a quick wind and dewpoint analyses on this pressure surface in regions where stations are far removed from the chosen isotherm. Otherwise where the isotherm is at or near a station location, the station report is used directly.

Once θ_k is chosen, the thermodynamic diagram is used to determine the unique temperature that θ_k corresponds to at the 700mb and 500mb levels. Trace those isotherms and transfer them to the θ_k map as 700mb and 500 mb isobars respectively. Complete the isentropic map by transferring mixing ratio and wind information from the 700mb and 500mb maps as was done for the 850mb level.

This technique allows meteorologists to construct isentropic maps, containing pressure, moisture and wind informa-

tion, on an operational basis. The method combines information from *three* separate pressure surfaces onto *one* isentropic surface, yielding a coherent depiction of the flow between pressure levels and thus a better diagnosis of the current weather situation (an important first step in any forecast). A word of caution however: because the data is restricted to three pressure levels, little information is gained utilizing this technique in a barotropic domain especially in the summer months when the 850mb and 700mb isobar could be over 1000 km apart. The technique should be applied to more baroclinic situations related to cyclogenesis, frontal bands and the jet stream, where significant vertical motion usually exists.

Examples of isentropic maps constructed solely from information on the standard 850mb, 700mb and 500mb pressure charts and the relationship of these charts with more conventional analyses are presented in Figures 5 through 8. The analyses attempt to isolate upward vertical motion by considering the advective term only (Eq. 4, Term 2), to depict the interaction within the lower, mid and upper troposphere, and to relate upward vertical velocity to the occurrence of precipitation and the position of jet cores.

CASE 1: 0000 GMT 7 March 1975: The 291K isentropic analysis (Fig. 5A) illustrates a significant northward moisture transport rising through the 850mb level from northeast Kansas to northern Illinois which then turns abruptly eastward from southern Minnesota to northern Wisconsin, flowing along the 700mb isobar. The region of upward vertical velocity (shaded region, Fig. 5C) coincides with the significant precipitation which extends from the thunderstorms in northeast Kansas to the moderate snows in northern Iowa through south-central Wisconsin into western lower Michigan. The sharp cut off of snowfall in southern Minnesota and central Wisconsin coincides with the cut off of the vertical motion at the 700mb level on this surface.

The upward vertical velocity is located within the left forward quadrant of the jet core which extends into southwest Missouri and the right rear quadrant of the jet core which extends from northeast Wisconsin into the New England region (Fig. 5C). The upslope motion on this isentropic surface appears to represent the rising branches of the "indirect" circulation of the southern jet and the "direct" circulation of the northern jet (see Appendix B).

CASE 2: 1200 GMT 16 February 1976: This case (Fig. 6) involves an open wave located in Kansas with a significant northward transport of moisture on the 297K isentropic surface flowing up through the 850mb level in northern Missouri and central Illinois. As in case 1, the upward vertical motion appears to be cut off at the 700mb level (Fig. 6A) where the moisture transport is directed more to the east. This corresponds with the northern boundary of the showers and thunderstorms which extend in a narrow band from northern Missouri through central Illinois into west-central Indiana (Fig. 6B). The region of upward vertical motion is located within the rising branch of the indirect circulation of a jet core to the southwest of the open wave and the direct circulation of the jet core to the northeast (Fig. 6C).

CASE 3: 12000 GMT 15 February 1876: The 291K surface (Fig. 7A) depicts upward vertical motion through the 850mb and 700mb levels from northeast Wisconsin and northeast lower Michigan to southern Canada. Within this

³The dewpoint on a pressure surface converts to a unique mixing ratio which can be easily obtained from a thermodynamic diagram.

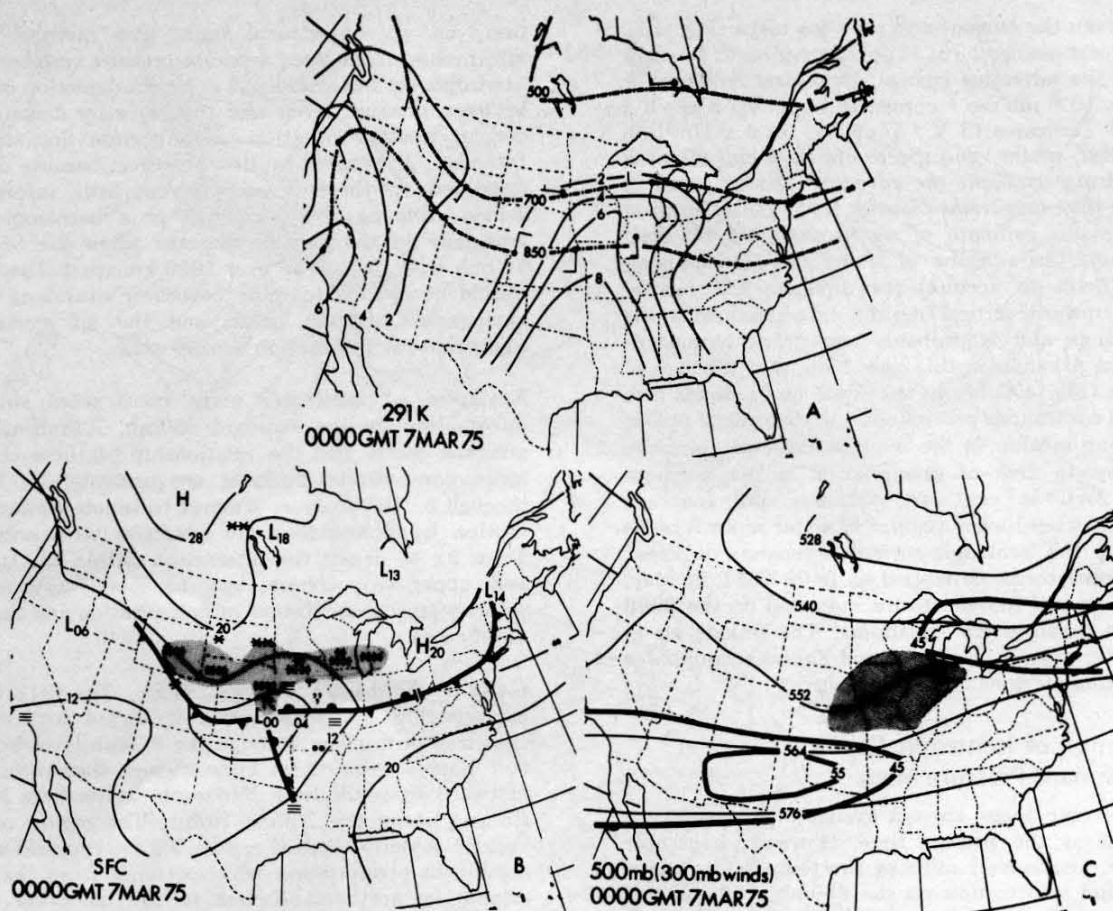


FIG. 5: A; 291K isentropic surface constructed by operational technique. Isobars (dot dashed) and mixing ratio g/kg (thin solid),
 B; surface analysis with continuous precipitation shaded,
 C; 500mb height (thin solid) and 300mb isotachs, m sec^{-1} (thick solid). Upward vertical velocity as estimated from advective term applied to 291K surface shaded, for 0000 GMT 7 March 1975.

region significant amounts of snow fell, northeast of an open wave located in northwestern Wisconsin (Fig. 7B). The distinct western boundary of the precipitation coincides with the sharp cut off of the upward vertical motion. The vertical motion is within the upward branches of the indirect circulation of a jet core located to the southwest of the surface low and the direct circulation of a jet core located to the northeast (Fig. 7C).

Cases 1 through 3 are similar in that they all involve an open wave with a relatively shallow closed circulation which apparently does not extend beyond the 850mb level. However, the distribution (with respect to the surface low) and areal extent of the precipitation vary in all three cases, a factor which is accounted for with the operational isentropic analysis technique.

CASE 4: 12000 GMT 21 February 1976: This case (Fig. 8) involves an occluded cyclone rather than an open wave. On the 291K isentropic surface (Fig. 8A) the moisture transport is northward and up through the 850mb level in northern Indiana and Ohio where it then splits into two branches. The branch that extends west then southwest, rises through the 700mb level from northeastern Wisconsin toward eastern Nebraska and reflects the cyclonic circulation which develops upward as the vortex occludes, while the other branch extends northeastward into southern Canada and New York. The isentropic analysis corresponds with the surface observations that indicate two separate areas of moderate precipitation, one to the northeast of the

occluded low and one to the southwest, west and northwest of the low (Fig. 8B), and with the infrared satellite picture (Fig. 9) which clearly illustrates these separate regions of continuous precipitation. As in the first three cases the upward vertical motion is within the left forward quadrant of a jet core to the southwest of the occluded center and the right rear quadrant of a jet core to the northeast of the center, and also extends along and to the east of the complex frontal zones associated with the storm.

5. Summary

The information provided by isentropic analysis, which is presently being ignored, could be a valuable asset to operational meteorologists for diagnosing current weather situations. Whether generated by a RAOB processing program or by the operational technique, isentropic maps provide a means for analysis on sloped surfaces that may extend from the boundary layer to the upper troposphere and therefore yield:

- 1.) a more coherent three dimensional depiction of moisture transport than can be attained using 850mb or 700mb analyses,
- 2.) a *simple* means for *directly* estimating vertical velocity dp/dt ,
- 3.) physical insight into the juxtapositions of the vertical motion field, moisture transport and the upper tropospheric jet stream.

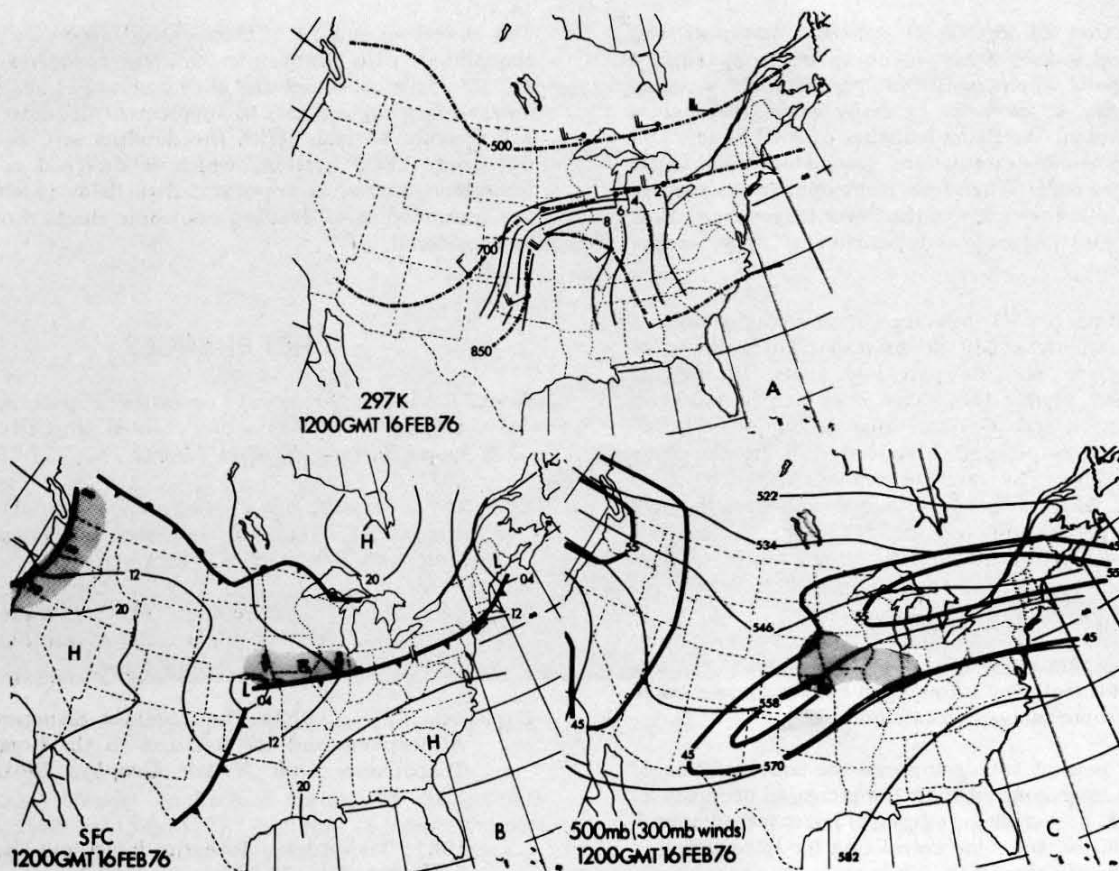


FIG. 6: The 297K isentropic analysis, surface analysis, and 500mb height, 300mb isotach analyses for 1200 GMT 16 Feb 1976. See caption for Fig. 5 for further details.

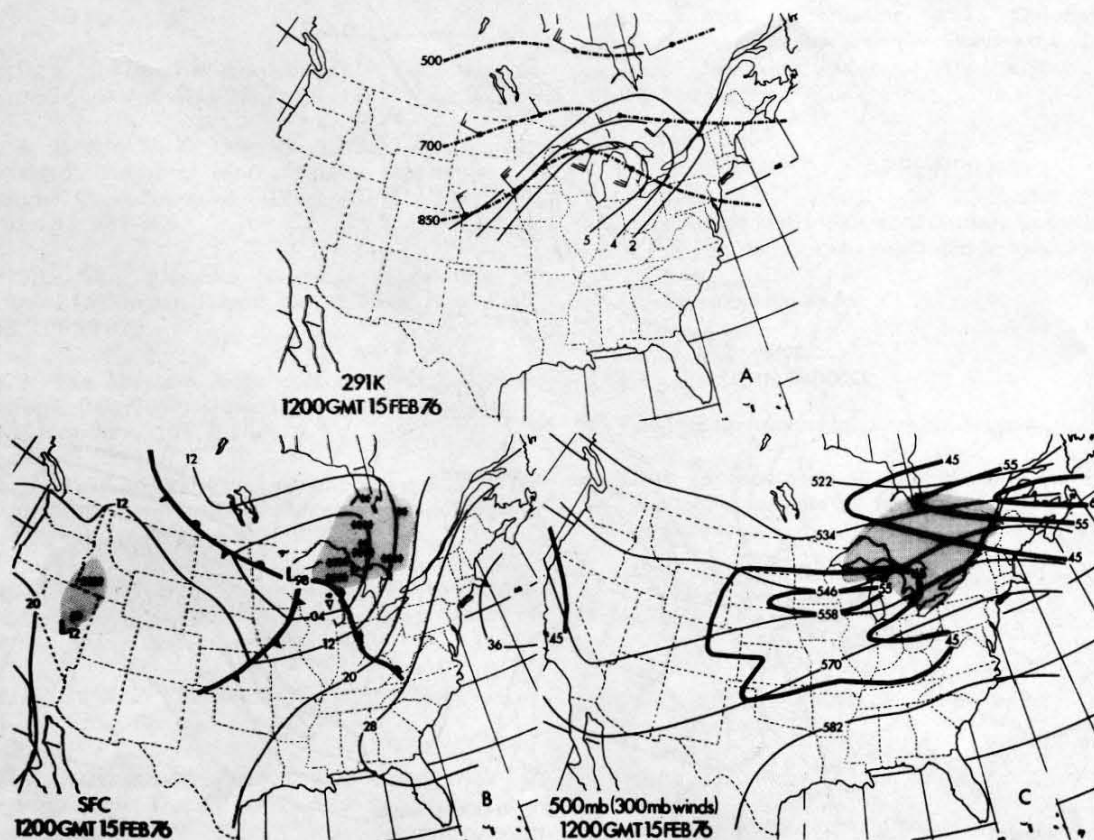


FIG. 7: The 291K isentropic analysis, surface analysis, and 500mb height, 300mb isotach analyses for 1200 GMT 15 Feb 1976. See caption for Fig. 5 for further details.

The combination of significant moisture transport and upward vertical motion analyzed on an isentropic surface delineates regions where significant precipitation is occurring or is likely to develop. In many cases, these areas appear to represent the rising branches of the "direct" and "indirect" transverse circulations associated with upper tropospheric jet cores. Therefore, isentropic analysis serves to better relate the response of the lower tropospheric flow patterns to the existence and position of upper tropospheric jet cores.

National Weather Service personnel have participated in a ten month advanced study program at the University of Wisconsin-Madison for the past two years. During the course of their study, they have been acquainted with isentropic analysis and the operational technique of generating isentropic maps, and have tested it in real time forecast situations. The reaction among them has ranged from a slight interest to a strong desire to have detailed isentropic maps available on an operational basis. This feeling echoes Oliver and Oliver's (1951) desire to have isentropic analysis reinstituted in the early fifties:

"Experienced forecasters and analysts, however, who had become familiar with the isentropic chart generally felt, and still feel, that a long step backwards was taken when isentropic analysis was abandoned.

"With the present trends towards the concentration of analysis in large centers, with the increased accuracy of radiosondes, and with the advent of improved communication facilities, the time seems ripe for reinauguration of the isentropic chart." (p. 726)

The increasing number of meteorologists who are becoming acquainted with isentropic analysis recognize it as a valuable diagnostic tool and they desire the transmission of detailed isentropic charts to supplement the current standard pressure analysis. With the development of the computerized AFOS system, which is designed to provide forecasters access to important data fields (Klein, 1976), the transmission of detailed isentropic charts should again be considered.

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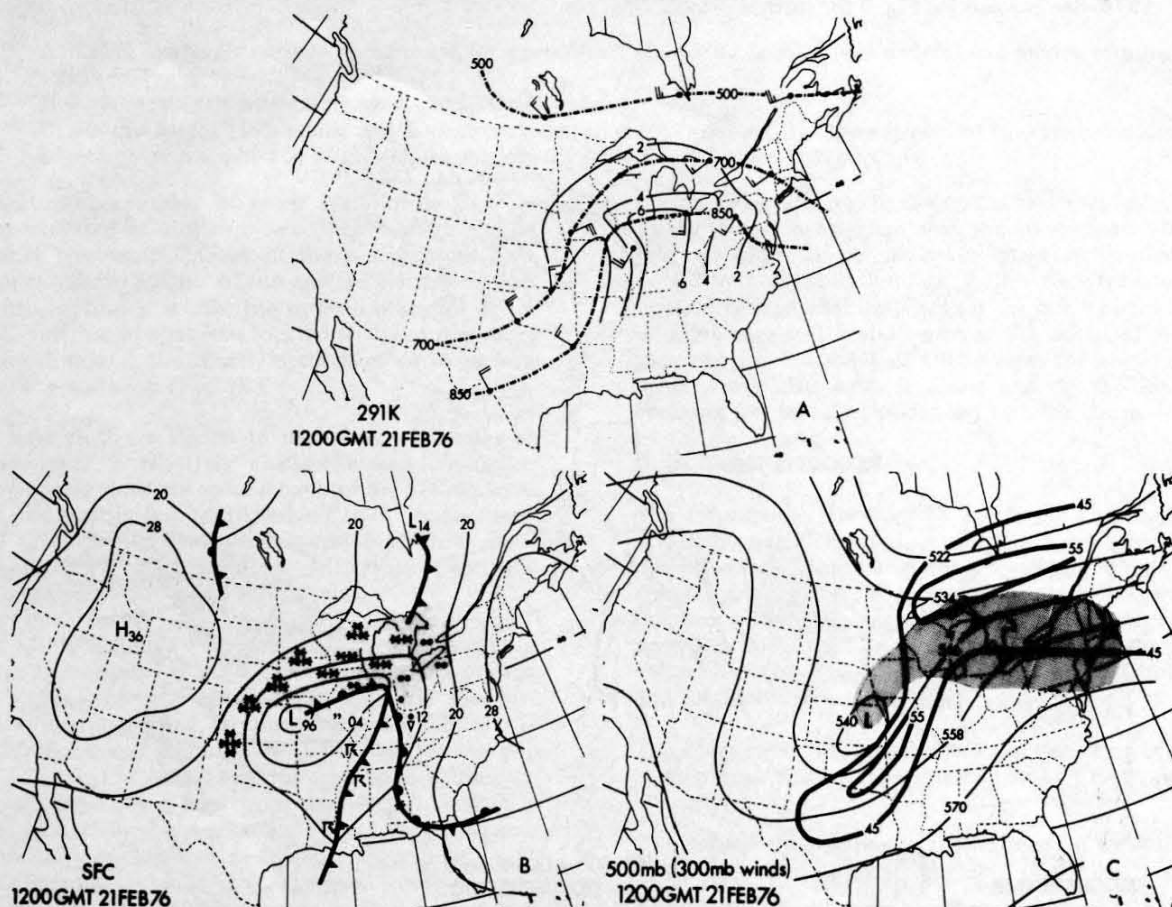


FIG. 8: The 291K isentropic analysis, surface analysis, and 500mb height, 300mb isotach analyses for 1200 GMT 21 Feb 1976. See caption for Fig. 5 for further details.

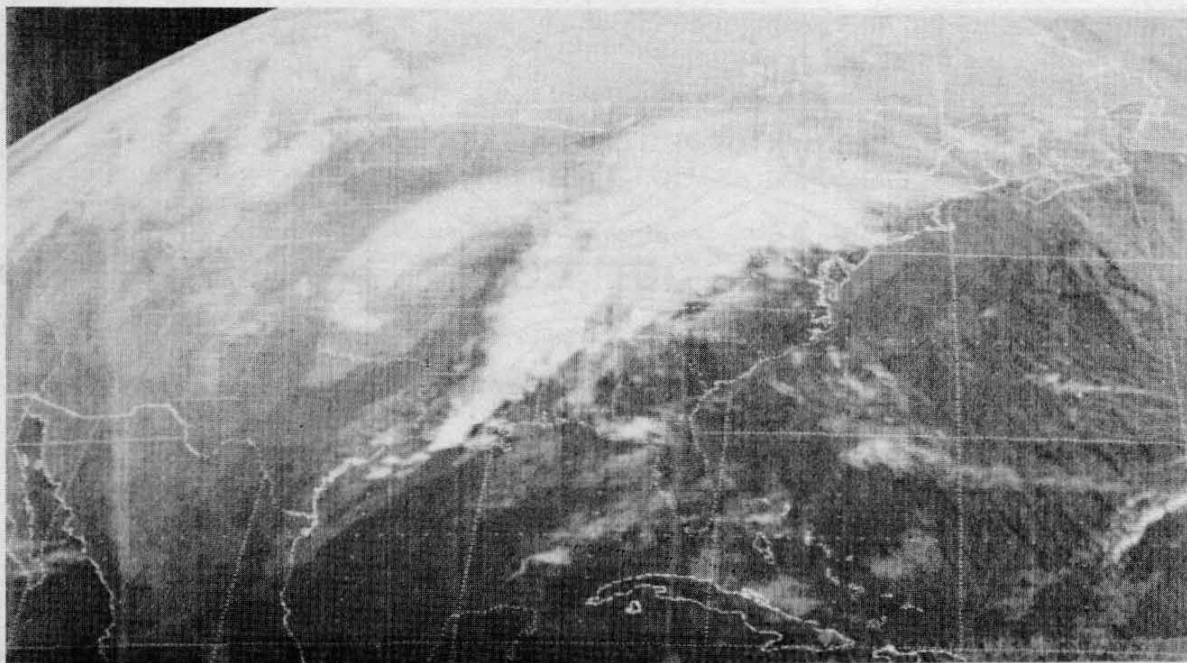


FIG. 9: Infrared satellite picture for 1430 GMT 21 Feb 1976.

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APPENDIX A

Basic mathematical definitions and relationships for the isentropic framework.

Poisson's equation

$$\theta = T \left(\frac{p}{p_{00}} \right)^K, \quad (A1)$$

defines the unique relationship between θ , T and P .

The Montgomery stream function (ψ) is derived on isentropic surfaces by vertically integrating the *hydrostatic equation*

$$\frac{\partial \psi}{\partial \theta} = c_p \left(\frac{\bar{p}}{p_{00}} \right)^{R/c_p}, \quad (A2)$$

with the surface value of ψ specified by

$$\psi_s = (c_p T_s + g z_s), \quad (A3)$$

where \bar{p} is the mean pressure in a layer, p_{00} equals the 1000mb reference pressure, c_p is the specific heat of dry air at constant pressure, R is the gas constant, T_s is the surface temperature, g is gravity and z_s is the surface elevation.

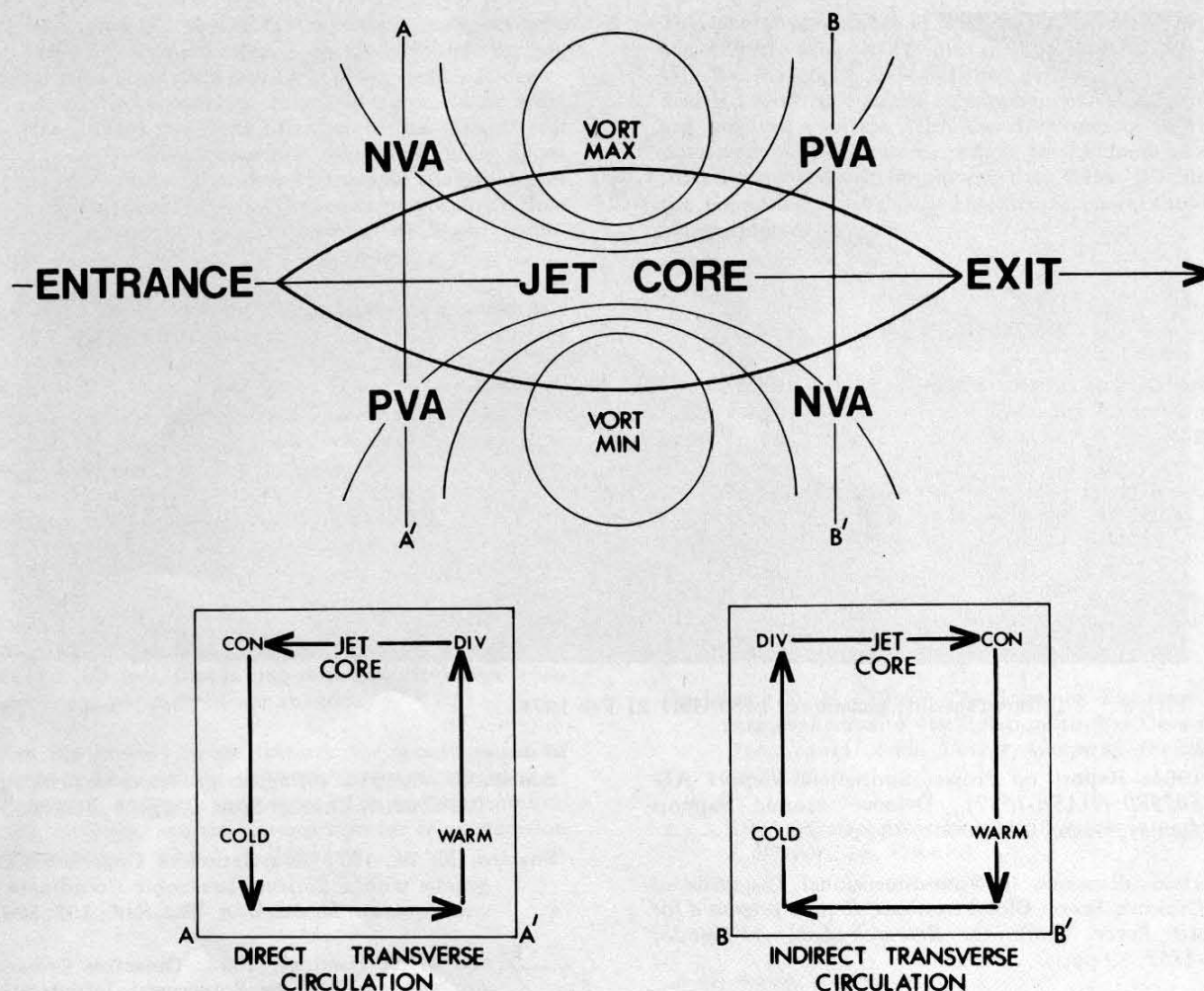


FIG. 10: Above: Schematic of zonal jet core, vorticity pattern and vorticity advection. Below: Cross sections through entrance (along A-A' line) and exit (along B-B' line) regions illustrating the direct and indirect transverse circulation.

The geostrophic wind vector is related directly to the gradient of ψ with $\vec{V}_g = 1/f (\mathbf{k} \times \nabla_\theta \psi)$. (A4)

The thermal wind vector is related to the pressure gradient by the expression

$$\vec{V}_T \equiv \frac{\partial \vec{V}}{\partial \theta} = \frac{R}{f} \left(\frac{p}{p_\infty} \right)^{(\kappa-1)} (\mathbf{k} \times \nabla_\theta p) \quad , \quad (A5)$$

where $\kappa = R/c_p$, $\nabla_\theta p$ is the pressure gradient, and \mathbf{k} is the unit normal vector along the vertical axis, and f is the Coriolis parameter.

The four cell vertical motion pattern can be related to vorticity advection (Fig. 10). The NVA at the jet-streak level in the cyclonic rear and anticyclonic forward quadrants coincides with convergence aloft and subsequent sinking motion. The PVA in the anticyclonic rear and cyclonic forward quadrants is related to divergence aloft and subsequent rising motion.

An important difference exists between the transverse circulations in the entrance and exit regions of a jet core. In the entrance region, cold air rises and warm air sinks converting kinetic energy to available potential energy. Thus the transverse circulation in the exit region is an "indirect" circulation (Johnson, 1970).

APPENDIX B

Transverse circulations associated with jet streaks

A basic characteristic of a jet streak with maximum winds exceeding the propagation rate of the core is the transverse circulations which develop in the entrance and exist regions of the jet core (Reiter, 1963; Cahir, 1971). As air parcels accelerate into the core, the ageostrophic components force a transverse circulation with rising motion in the left (cyclonic) quadrant and sinking motion in the right (anticyclonic) quadrant (Fig. 10).

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