DETERMINING DIVERGENCE AND ASSOCIATED RATE OF PRECIPITATION USING THE 3-HOURLY TENDENCY AND PRESSURE FIELDS

Charles H. Pierce
Retired, National Weather Forecast Office, Boston, Massachusetts

ABSTRACT

Quantitative precipitation forecasts are transmitted to the field forecasters by the National Meteorological Center once every 6 hours. However, occasions may arise when the meteorological situation may have changed and the meteorologists would like to revalue the predicted amount. Other papers have been written relating the rate of precipitation to vertical motion through the field of divergence. However, considerable time is consumed before the precipitation amount can be computed. The technique shown here is simple. After making a few assumptions, the rate of precipitation can be quickly determined at one locality from parameters taken directly from the surface weather chart or the hourly teletype sequence. This is accomplished through the use of Showalter's rate of precipitation by substituting for vertical velocity the divergence determined by the horizontal change in gradient of the 3-hourly pressure tendency and the horizontal change of the pressure gradient plus a coriolis-density term.

1. INTRODUCTION

In this day of numerical weather predictions and electronic computers, methods of forecasting precipitation amounts have become rather sophisticated. However, these forecasts are made once every six hours. Between these forecasts, the meteorological situation can change, and the forecaster is faced with re-evaluating the amount of precipitation predicted for his and other weather stations in the forecast area. Although the technique for determining the quantitative precipitation amounts described here is simple and less exact than the machine method, it can be used at any time by using the three hour pressure changes and the pressures in a grid consisting of a central station and four surronding stations approximately equidistant from the central station.

In the *Monthly Weather Review* of January 1957, K.R. Peterson's "Precipitation Rate as a Function of Horizontal Divergence" contains a graph which gives the rate of precipitation for a six-hour period. He uses as his arguments (variables): surface convergence assumed to be decreasing linearly with height to zero at 4.5 km, and the 1000-mb or the 850-mb temperature. Although Peterson pointed out that Thompson and Collins determined the horizontal velocity divergence for several levels by using the Bellamy technique, he did not offer any method of finding the surface convergence or divergence. It is the purpose of this paper to describe a method of roughly determining the surface convergence by making use of the 3-hourly pressure tendencies and the pressure field available from either the latest weather chart or from the hourly teletype sequences.

2. TECHNIQUE

Surface divergence is related to the product of the local change of relative vorticity with time and the reciprocal of the absolute vorticity. This assumes that the surface wind everywhere is geostrophic, that there is no advection of vorticity at the surface, that there is no conversion of vorticity from the horizontal axis to the vertical and there are no frictional or solenoidal effects. Since the local change of surface vorticity is related to the change in the horizontal gradient of pressure change, and the absolute vorticity is related to the change in horizontal pressure gradient, it follows that the surface convergence is also related to these propoerties.

To determine the change in gradient of pressure change and pressure, a grid is marked off with the center of the grid as the point where divergence or convergence is to be determined. The three-hour pressure changes at the four corners of the grid are added together. From this sum, four times the pressure change at the center of the grid is subtracted. The same procedure is done for pressures. Divergence is the negative ratio of the differences of the pressure change and the differences in pressure plus another term in the denominator. This term, the denominator is the product of the square of the coriolis parameter, surface density, and the square of the grid distance. (See equation 6 in the Appendix.)

After the surface convergence (negative divergence) has been calculated, the vertical velocity of the layer must be determined. Through the equation of continuity, the vertical velocity at the top of the layer is related to the surface convergence, if we assume a linear change in convergence from the surface to the level of non-divergence at 500 mb, or about 5500 meters.

Showalter determined an equation based on a development by Fulks, for the rate of rainfall using the vertical velocity of the layer and the density at the bottom of the layer and the difference between mixing ratio at the bottom of the layer and at 500 mb. Since we are dealing with a saturated atmosphere, the 850-mb temperature, or the surface dewpoint taken close to the warm front describes the

density and mixing ratio at the bottom of the layer, (assumed to 950 mb), and the mixing ratio at the 500-mb level. Thus the 6-hourly rate of precipitation for a saturated atmosphere may be determined from knowing the surface convergence and the 850-mb temperature (or, if not available, the surface dewpoint close to the warm front.) The relationships between the 3-hourly convergence, mean vertical velocity, and 6-hour rainfall is shown in the following table for an atmosphere saturated from 950 to 500 mb and a temperature of 5°C at 850 mb at a latitude of 45°.

A graph has been included with this paper to aid in quick determination of 6-hour rainfall for other values of convergence and 850-mb temperatures. This graph has been constructed for an atmosphere saturated from 950 to 500 mb. The ordinate (Figure 1) gives the 850-mb temperature (or the surface dewpoint), the abscissa shows the 3-hourly convergence, and the curved lines are the 6-hourly precipitation. The term $f^2 \rho\ d^2$ which must be added to the denominator for figuring convergence is shown at the right of the graph. This is for every 5 degrees of latitude from 30°N to 50°N. Density is figured at 1000 mb using 20°C for the lowest latitude, 15°C for the middle latitudes and 10° and 5°C respectively for the two highest latitudes. The grid distance used for this table was 150 nautical miles. For example, at 45°, f is 1.03×10^{-4}, the density ρ, is 1.25 kg m^{-3} and 150 nautical miles are 2.78×10^5 meters. Converting from pascals to millibars $f^2 \rho\ d^2$ becomes 10.2.

Here is an example of how to use the graph (Figure 1): if the 3-hourly tendencies at the four corner points 150 nautical miles from your station are: -2.5, -2.6, -0.5, and -0.6 and at your station -2.0, the numerator would be 1.8 mb. The sea level pressure at the same points are: 1011, 1012, 1016, and 1017 mbs. At your station, located at 45°N, the pressure is 1015 mb. The denominator would be 4056-4060 + 10.2 or 6.2 mb. Since the sign of the divergence value is negative, hence the convergence (negative divergence) would be +0.29. If the 850-mb temperature is 5°C for the saturated air mass, the rate of rainfall for 6 hours, according to the graph, would be 0.4 inches.

One must be sure that the sign for the quotient is correct. If the average pressure falls for the corner points are larger than the pressure fall at your station, the numerator would be negative and, since the sign in front of the divergence equation is negative, the quotient would be positive indicating divergence. By the same token, if the pressure rises at the surronding points average larger than at your station, the numerator would be positive, making the quotient negative, indicating convergence. Except for rare cases, the denominator is always positive.

For the best results in determining the 6-hourly precipitation from convergence, the calculation should be made when the low pressure system is southwest of your station and moving eastward, or south of your station and moving northeastward. Make the computation soon after the steady precipitation has begun or when the pressure has started to fall rapidly. This method will not work very well if the lowest pressure is about to pass over the station. After the precipitation has been falling for 3 hours, you may want to compute what the precipitation should be for 6 hours. At this point, the method described herein may be used, but it would be quicker and probably just as accurate to arrive at your estimate by doubling the rainfall for the past three hours.

In some cases, such as fast moving systems, the precipitation may not continue for 6 hours. In that event, the hourly rate should be determined by dividing the computed rate by 6. Your estimate would then be obtained by multiplying by the number of hours precipitation is likely to last.

Except as a first approximation, this method is not too useful for areas where terrain plays a major role in the production of precipitation. It should not be used for con-

TABLE I

Convergence per 3 hours	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Vertical Velocity m/3-hours	215	429	644	858	1073	1287	1502	1716	1931
6-hour rainfall, inches	0.14	0.28	0.42	0.56	0.70	0.84	0.98	1.12	1.26

vective situation or when the warm sector air is unstable.

The grid distance, distance from the center of the grid to one of the corners, used here was 150 nautical miles. This distance probably should be no greater, but a shorter distance can be used. If a shorter distance is used, the term $f^2\rho\ d^2$ must be recalculated.

3. DISCUSSION

A cursory test was made for 25 situations when rain occurred in the northeast and midwest. In 18 of the 25 cases, the predicted precipitation amount was too small. The average error was 0.10. The correlation coefficient between predicted and observed rain was 0.63.

An interesting recent check made from the pressure changes at Boston and surronding grid points at 06 GMT on 17 September 1976, indicated surface divergence of 0.24 per three hours. This would produce negative vertical velocity and theoretically no rain. The QPF for the same time was 0.12 inches. The actual rain was 0.01 inches. Quite often, the quantitative precipitation predictions made by the computer will be close to verifying correctly, but when developments are not going the way the computer indicates, calculations made by using the graph in this paper will serve as an interesting check.

4. REFERENCES

Bellamy, J.C. , 1949: Objective Calculations of Divergence. Vertical Velocity, and Vorticity. *Bulletin of the American Meteorological Society*, V 30, pp 45-49.

Fulks, J.R., 1935: Rate of Precipitation from Adiabatically Ascending Air. *Monthly Weather Review*, V 63 N 10, pp291-294.

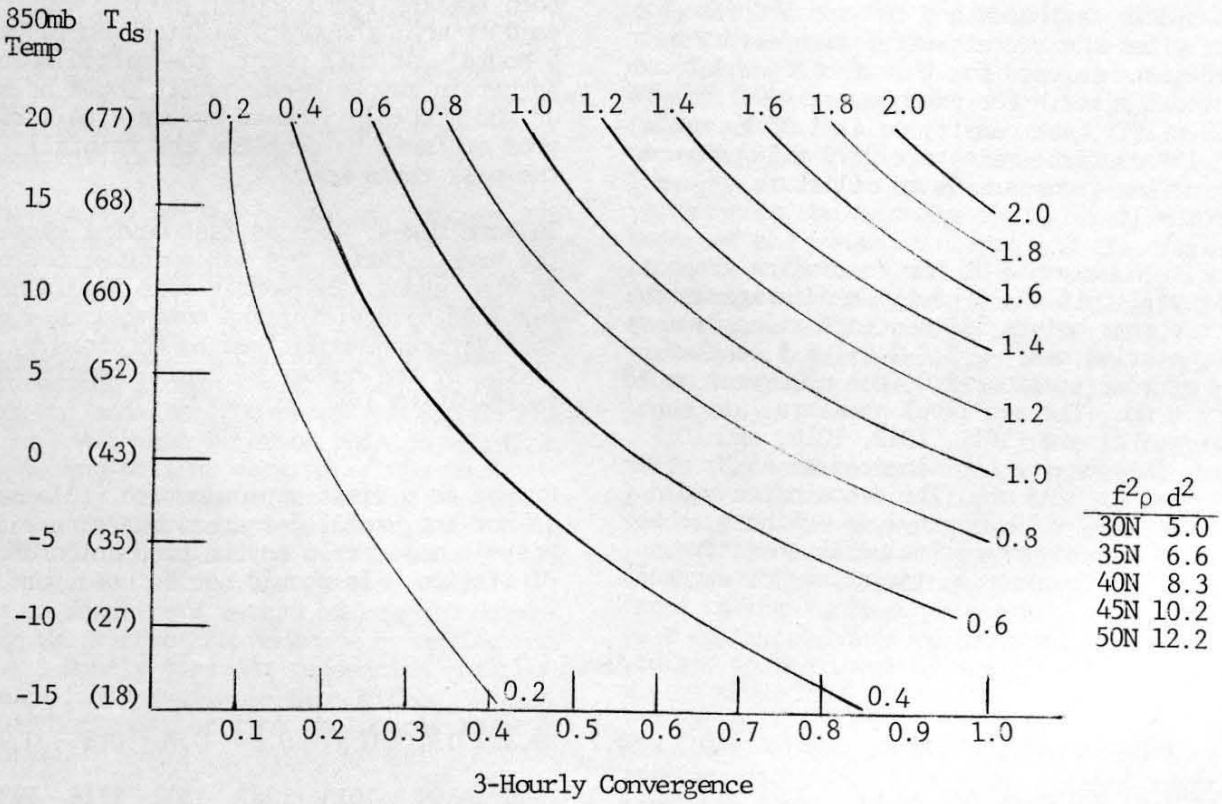

Figure 1. 6-Hour Precipitation (inches)

Peterson, K.R. 1957: Precipitation Rate as a Function of Horizontal Divergence. *Monthly Weather Review*, V 85 N 1, pp 9-10.

Showalter, A.K., 1944: Rates of Precipitation from Pseudo-Adiabatically Ascending Air. *Monthly Weather Review*. V 72, N 1. pg 1.

Thompson, J.C. and Collins, G.O., 1953: A Generalized Study of Precipitation Forecasting. Part I: Computation of Precipitation from the Fields of Moisture and Wind. *Monthly Weather Review*, V 81. N 4. pp 91-100.

5. APPENDIX

The horizontal surface divergence is given by the formula:

$$(1) \quad \nabla \cdot \mathbb{V}_s = - \frac{1}{\eta} \frac{\partial \zeta}{\partial t}$$

where $\mathbb{V}_s$ is the surface geostrophic wind, ζ is the relative vorticity, η is the absolute geostrophic vorticity and t is time. The formula for the partial derivative of vorticity with time is:

$$(2) \quad \frac{\partial \zeta}{\partial t} = \frac{1}{f\rho} \nabla^2 \frac{\partial p}{\partial t}$$

and for absolute vorticity it is:

$$\eta = \frac{1}{f\rho} \nabla^2 p + f$$

where ρ is density, f is the coriolis parameter, p is pressure and ∇^2 is the Laplacian operator.

Substituting into equation (1), we arrive at;

$$\nabla \cdot \mathbb{V}_s = - \frac{\frac{1}{f\rho} \nabla^2 \frac{\partial p}{\partial t}}{\frac{1}{f\rho} \nabla^2 p + f}$$

Simplifying:

$$\nabla \cdot \mathbb{V}_s = - \frac{\nabla^2 \frac{\partial p}{\partial t}}{\nabla^2 p + f^2 \rho}$$

Now by using a finite difference form of the equation, surface divergence becomes:

$$\nabla \cdot \mathbb{V}_s = - \frac{\Delta p_1 + \Delta p_2 + \Delta p_3 + \Delta p_4 - 4\Delta p_o}{p_1 + p_2 + p_3 + p_4 - 4p_o + f^2 \rho d^2}$$

where Δp is the 3-hourly tendency, p is the pressure at the grid points and d is the grid distance.

To convert surface divergence or convergence to vertical velocity, use is made of the equation of continuity and vertical velocity is shown by:

$$w_z = - \frac{\rho_m}{\rho_z} 1/2 \{\nabla \cdot v_s + \nabla \cdot \mathbb{V}_z\} z$$

where w_z if the vertical velocity at height z, ρ_m if the mean density, ρ_z is the density at height z and $\mathbb{V}$ if the horizontal velocity vector.

If the level of non-divergence is 5500 meters $\frac{\rho_m}{\rho_z}$ is 1.3 and the mean velocity v_m is 0.6 of w_z, we have:

$$v_m = - 2145 \nabla \cdot \mathbb{V}_s$$

Using Showalter's equation for determining the rate of precipitation and using the mean vertical velocity, the equation for the 6 - hour rainfall becomes:

$$I = 0.08 \{w_m \rho_o (x_o - x_1)\}$$

where I is the rainfall for 6 hours in inches, ρ_o is the density and x_o is the mixing ratio at the bottom of the layer and x_1 is the mixing ratio at the top of the layer.

Finally using equation (8) in (9), the rainfall for 6-hours becomes:

$$I = -170 \times 10^3 \nabla \cdot \mathbb{V}_s \rho_o (x_o - x_1)$$