**computer**

# A COMPUTER PROGRAM FOR CONVECTIVE PARAMETERS

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## 1. INTRODUCTION

This program computes, for a given parcel of air, the Lifted Condensation Level (LCL), Convective Condensation Level (CCL), Convective or Critical Temperature ($T_c$), the energy in the positive area of the sounding (if it exists) in joules/gram, and the height in millibars and feet above ground level (AGL) at the point where the positive area again becomes negative. The parcel of air is lifted dry-adiabatically to the LCL, then pseudo-adiabatically thereafter. The actual environmental sounding is modeled within the program, from input data, so that the temperature of the parcel and the environment can be compared at appropriate intervals. This technique allows the positive area (energy) of the parcel to be calculated. Only eight input variables, easily obtained from RAOB data, are necessary to run the program.

The program was written in FORTRAN using the development computer facility at Western Region Headquarters, National Weather Service (WRH NWS) and is run daily at Great Falls WSFO on the AFOS computer. The output data are useful in public and aviation forecasting. In particular, the energy computation of the positive area is an excellent indicator of the strength of potential convective activity.

## 2. METHODS AND EQUATIONS

The temperature at the LCL is computed using a version of Haurwitz' equation where $t_d$ is the dew-point temperature and D is the dew-point depression

```
C  CONVECTIVE PARAMETERS PROGRAM
C  K. MIELKE ... FEB 1979 ... WSFO GTF.
   EXTERNAL POSAREA LCLCCL HGTAGL DIMEN-
   SION TINPUT (10.8)
   CALL OPEN (5, "DP1 TSTMIN", 2, IER, 512)
   CALL OPEN (7, "DP1 TSTMOUT", 2, IER, 512)
   READ (5, 1) N, ((TINPUT (I,J), J=1,8), I=1,6), XM,
   DA, YR
1  FORMAT (/T6.I1,/6 (/T6, F4.0, T13, F4.1, T20, F5.1,
   T28, F5.1, T36, F5, 1, T44, 1 F5.1, T52, F5.1, T60,
   F5.1), /T13, F3, 0, T20, F3, 0, T28, F5, 0)
   00 50 1=1, N
   A = TINPUT (I, 1)
   B = TINPUT (I, 2)
   C = TINPUT (I, 3)
   D = TINPUT (I, 4)
   E = TINPUT (I, 5)
   F = TINPUT (I, 6)
   G = TINPUT (I, 7)
   H = TINPUT (I, 8)
C  SATURATION VAPOR PRESSURE AT 850 MB
   E1 = 6.11* (10** (7.567* C/(239.7+C)))
C  SATURATION MIXING RATIO AT 850 MB
   W = .622*E1/850
C  SATURATION VAPOR PRESSURE AT SFC
   E2 = W*A/.622
C  SURFACE DEWPOINT T3 = (ALOG10 (E2/6.11))*
   239.7/(7.567 - ALOG10 (E2/6.11))
C  DEWPOINT DEPRESSION
   D1=B-T3
C  REDO DWPT IF> T
   IF (B-T3) 2, 2, 4
2  T3 = B-1
   D1 = 1
C  ACTUAL LAPSE RATE 300-200MB
4  Z = (H-G) /100
C  ACTUAL LAPSE RATE 500-300MB
   Z1 = (G-F) /200
C  ACTUAL LAPSE RATE 700-500MB
   Z2 = (F-E) /200
C  ACTUAL LAPSE RATE 850-700MB
   Z3 = (E-D) /150
C  ACTUAL LAPSE RATE SFC-850MB
   Z4 = (D-B) / (A-850)
C  CALL SUBROUTINE TO DETERMINE T, P AT LCL
   AND CCL
   CALL LCLCCL (T3, D1, A, B, Z4, D, Z3, E, Z2, F,
   Z1, G, T1, P1, BB, T, P)
C  ROUND PRESSURE TO NEAREST 10 MB
   P = FLOAT (IFIX (P + 0.5))
```

("t" indicates $^oC$ and "T" indicates $^oK$) (Inman 1969).

$$t_{LCL} = t_d\,(.212 + .001571\,t_d - .000436\,t)D \quad (1)$$

The pressure at the LCL is calculated from Poisson's Equation in the form:

$$P_{LCL} = P(T_{LCL}/T_o)^{Cp/Ra} \quad (2)$$

where $T_o$ = initial temperature ($^oK$) and P = initial pressure (mb).

where $T_o$ = initial temperature ($^oK$) and P = initial pressure (mb).

In a saturated atmosphere, the lapse rate is not constant. An equation for the moist adiabatic lapse rate given by Haurwitz, as shown by Berry, et al. (1945), is utilized (see Appendix I for symbols).

$$\gamma = \frac{R_aT}{C_pP} \cdot \frac{P+(.622L_ve_s/R_aT)}{P+(.622L_v^2e_s/C_pR_2T^2)} \quad (3)$$

The assumption $e \simeq e_s$ is used.

The saturation vapor pressure is computed using Teten's equation (Berry, et al., 1945). This equation assumes the presence of liquid water in the atmosphere below $0^oC$.

$$e_s = 6.11\,(10^{at/(b+t)}) \quad (4)$$

where a = 7.567 and b = 239.7.

The latent heat of vaporization (Lv) is linearly interpolated between its value at $30^oC$, which is 2428.45 j/g, and its value at $-50^oC$ which is 2633.05 j/g.

The CCL is normally defined (on a thermodynamic diagram) as the intersection of the saturation mixing ratio line through the surface dew point and the

```
C  REASSIGN P, T CCL
   P2 = P
   T2 = T
C  CONVECTIVE TEMP
   B1 = BB-1
C  CALL SUBROUTINE TO DETERMINE POSITIVE
   AREA
   CALL POSAREA (T, P, Z3, Z2, Z1, Z, A1, TT, PP)
C  CALL SUBROUTINE TO DETERMINE HGT AGL OF
   POS AREA
   CALL HGTAGL (D, TT, A, PP, Y)
   IF (P2-500) 20, 20, 22
20 A1 = 99.99
22 IF (I.GT.1) GOTO 55
   WRITE (7,38) XM, DA, YR
38 FORMAT (2X, ":", /2X, ":", /10X, "CONVECTIVE
   OUTPUT", 4X, F3.0, "/", F3.0, "/", F5.0,
1  /10X, "..... 99.99 = CCL ABV 500 MB ... POS AREA
   NOT MEANINGFUL .....",//)
   WRITE (7,40) T1, P1
40 FORMAT (2X, "TEMP AT LCL = ", F5.1,/2X, "PRESS
   AT LCL = "F5.0)
   WRITE (7,42) T2, P2
42 FORMAT (2X, "TEMP AT CCL = ", F5.1,/2X, "PRESS
   AT CCL = "F4.0)
   WRITE (7,44) B1
44 FORMAT (2X, "CONVECTIVE TEMP = ", F3.0)
   WRITE (7,46) TT, PP
46 FORMAT (2X, "TEMP TOP POS AREA = ", F5.1,/2X,
   "PRES TOP POS AREA = ", F4.0)
   WRITE (7,48) A1, Y
48 FORMAT (2X, "POS AREA (J/G) = ",F5.2, /2X, "HGT
   AGL TOP POS AREA = ",F8.0)
   GOTO 50
55 WRITE (7,60) I, A1
60 FORMAT (/2X, "POSITIVE AREA AT SITE
   NUMBER",I3," = ", F5.2)
50 CONTINUE
   WRITE (7,70)
70 FORMAT (////15X, "*", //15X, "*", //15X, "*",
   //15X, "*", //15X, "*")
   CALL CLOSE (5, IER)
   CALL CLOSE (7, IER)
   END
```

## SYMBOLS

$e$ = vapor pressure of water (mb)

$e_s$ = saturation vapor pressure of water (mb)

$w$ = mixing ratio

$c_p$ = specific heat at constant pressure for dry air 1.003 j/g$^o$K

$R_a$ = gas constant for dry air .287 j/g$^o$K

$R_v$ = gas constant for moist air .461 j/g$^o$K

$L_v$ = latent heat of vaporization (j/g) actual sounding. The CCL can also be computed without the use of a diagram. The CCL in this program is determined in a program loop which repeats the computation of the pressure and temperature for the LCL by incrementing the initial temperature and dew-point spread by $1^{o}$ during each iteration. When the temperature at the iterated LCL equals the environmental temperature, the CCL is then located.

The incremented surface temperature at this point is equal to the convective or critical temperature.

The surface dew point used in this program is not the observed value. Rather, a derived value is calculated from the 850-mb dew point. The 850-mb dew point is more conservative and often more representative of the low-level moisture. The surface dew point used in all calculations in this program is developed as follows.

Teten's equation (4) is used to obtain the saturation vapor pressure at 850 mb. From this, the saturation mixing ratio at 850 mb is found.

$$w_s^{850} = (.622\ e_s^{850})/850 \qquad (5)$$

This is converted to a saturation vapor pressure at the surface

$$e_s^{sfc} = (w_s^{850}\ P_o)/.622,\ P_o = \text{sfc Press} \qquad (6)$$

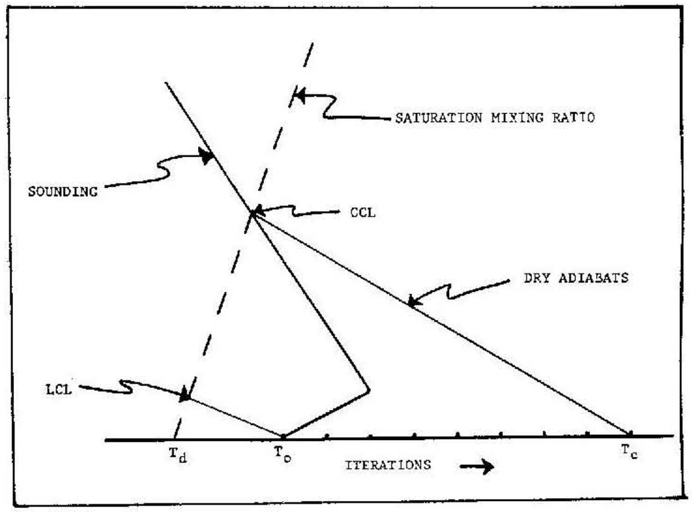
Then Teten's formula is rederived to solve for $t_d$, which gives the surface dew point when the above parameters are included.

$$t_d^{sfc} = \frac{(\mathrm{LOG}_{10}(e_s^{sfc}/6.11))\ 239.7}{7.567 - \mathrm{LOG}_{10}(e_s^{sfc}/6.11)} \qquad (7)$$

The actual sounding is modeled from the surface, 850-mb, 700-mb, 500-mb, 300-mb, and 200-

$\gamma$ = computed moist adiabatic lapse rate ($^{o}$K/mb)

dT/dP = environmental lapse rate ($^{o}$K/mb)




```
C   SUBROUTINE TO DETERMINE T, P AT LCL AND
    CCL
    SUBROUTINE LCLCCL (T3, D1, A, B, Z4, D, Z3, E,
    Z2, F, Z1, G, T1, P1, BB, T, P)
    N = 0
    BB = B
    DD = D1
1   T = T3 - (.212 + .001571*T3 - .000436*BB) *DD
    P = A* ((T + 273) / (BB + 273)) ** 3.5
    IF (N) 2, 2, 4
C   T LCL
2   T1 = T
C   P LCL (ROUNDED TO 10MB)
    P1 - FLOAT (IFIX (P + 0.5))
C   INCREMENT SFC T AND SFC DWPT SPRD BY 1
    FOR ITERATION
4   BB = BB + 1
    DD = DD + 1
C   CHECK P TO DETERMINE WHICH LAPSE RATE
    TO USE
    IF (P-850) 6, 6, 8
6   IF (P - 700) 10, 10, 12
10  IF (P-500) 14, 14, 16
14  GOTO 18
8   X = D - Z4* (P-850)
    GOTO 20
12  X = F - Z3* (P-700)
    GOTO 20
16  X = F Z2* (P-500)
    GOTO 20
10  X = G-Z1*
(P-300)
20  IF (T-X) 22, 24, 24
22  N = N + 1
    GOTO 1
24  RETURN
    END
```

mb temperatures, which are input initially. At the CCL both the temperature of the parcel ($T_p$) and the temperature of the environment ($T_e$) are equal. From that point upward, a new $T_e$ and $T_p$ are computed at each $\Delta P$ by finite difference techniques:

$$Tp_2 = Tp_1 + \Delta P \gamma m \qquad (8)$$
$$Te_2 = Te_1 + \Delta P \, (dT/dP(x))$$

where x = sfc to 850, 850 to 700, 700 to 500, 500 to 300, 300 to 200

As the parcel is lifted, its pressure is compared against the 5 above levels to determine which environmental lapse rate to use, in order that the sounding may be approximated at that point.

Computation of the positive area (energy) is accomplished using the coordinates of the Emagram (Berry, et. al, 1945; and Hess, 1959). On this diagram, area represents energy. The positive area is computed after each iteration ($\Delta P$) as illustrated.

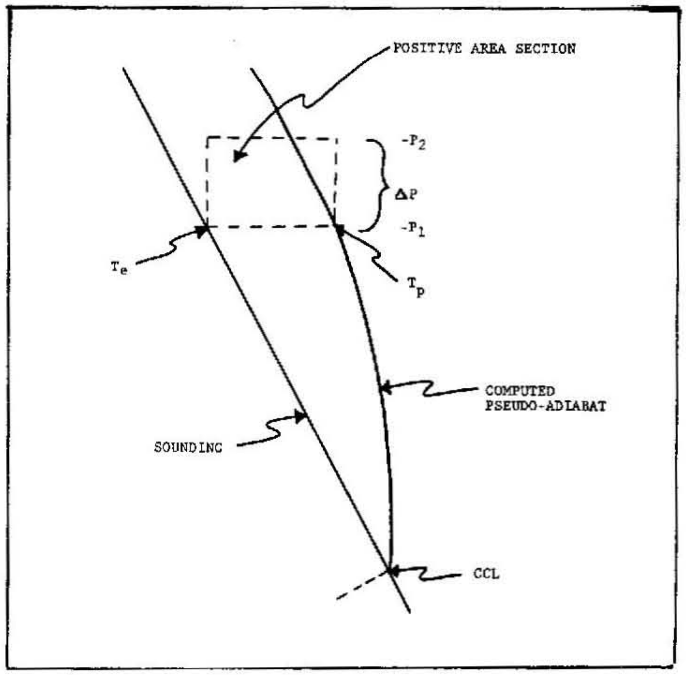
Positive Area =

$$(Ra\,(Tp-Tc))\,(Ln\, P_2/P_1)\ (j/g)$$

For sufficiently small $\Delta P$, the positive area can be quite closely approximated. The positive area is computed and accumulated until $T_p < T_e$. At this point, the iteration is terminated. The temperature and pressure at that level are output, along with the accumulated positive area in j/g.

The height in feet AGL is also computed at the point where $T_p < T_e$ using the Hypsometric Equation

$$\Psi_2 - \Psi_1 = -R_a^* \,\overline{T^*}/9.8\ Ln\,(P_2/P_1) \qquad (9)$$




```
C   SUBROUTINE TO DETERMINE POSITIVE AREA IN
    SOUNDING (J/G)
    SUBROUTINE POSAREA (T, P, Z3, Z2, Z1, Z, A1,
    TT, PP)
    X = T
    A1 = 0
    TT = T
    PP = P
C   LATENT HEAT OF CONDENSATION
C   SATURATION VAPOR PRESSURE
1   E4 - 6.11* 10** ((7.567 *TT) / (239.7 + TT))
    Z0 = 2428.45 + (303 - (TT + 273)) * (204.60) / 80
C   MOIST ADIABATIC FORMULA - NEXT 3 STATE-
    MENTS
    ZM1 = PP + (.622)*Z0*E4 / ((.287) * (TT + 273))
    ZM2 = PP + (.622) * (E4* Z0**2) / ((1.003) * (.461) *
    (TT + 273) ** 2)
    ZM3 = (.287) * (TT + 273) / (1.003 * PP)
    ZM4 = ZM3* ZM1/ZM2
C   NEW T (ADIABATIC EQ)
    TT = (-ZM4) * 10 + TT
C   DECREASE P BY 10 MB
    PP = PP - 10
C   CHECK P TO DETERMINE WHICH ENVIR LAPSE
    RATE TO USE
    IF (PP - 700) 2, 2, 4
2   IF (PP - 500) 6, 6, 8
6   IF (PP - 300) 7, 7, 12
7   GOTO 13
4   X = Z3 * 10 + X
    GOTO 14
C   USE (600/PP) TO NORMALIZE DT/DP
8   X = Z2* (600/PP) * 10 + X)
    GOTO 14
C   USE (400/PP) TO NORMALIZE DT/DP
    X= Z1 (400/PP)*10+X
    GOTO 14
C   USE (250/PP) TO NORMALIZE DT/DP
13  X = Z * (258/PP) * 10 + X
C   COMPUTE POSITIVE AREA
```

where

$\Psi$ represents geopotential meters, T represents the virtual temperature of the layer between $P_1$ and $P_2$, and $R_a^* = 287.04$.

The difference between the virtual temperature and the actual temperature of a parcel of air is usually less than $1^{o}K$. For this reason, the assumption $\bar{T}^* = \bar{T}$ was used. $\bar{T}$ is defined as the average temperature between the top of the positive area and 850 mb.

Beginning at the surface, $\Psi_1 = \Phi$ therefore,

$$\Psi_2 = -R_a^* \bar{T}/9.8 \; Ln \, (P_2/P_1) \qquad (10)$$

which is the height of the positive area in geopotential meters. This is converted to geopotential feet and output.

```
14 A2 = (TT-X) * (.287) * (ALOG ((PP + 10) /PP))
C  IF A IS NEG, TOP OF POS AREA REACHED
   IF (A2) 18, 16, 16
C  ACCUMULATE POS AREA (J/G)
16 A1 = A2 + A1
   GOTO 1
18 RETURN
   END

C  SUBROUTINE TO DETERMINE HGT AGL OF
   POSITIVE AREA
   SUBROUTINE HGTAGL (D, TT, A, PP, Y)
   T8 = (D + TT) / 2 + 273
   Y = (ALOG (A/PP)* T8 * (287.04) / 9.8) * (3.28)
   RETURN
   END
R
```

## 3. PROGRAM OPERATION

The program is initiated and data are entered from the AFOS console. The program resides on a magnetic diskette (storage device), along with an input and output file at Great Falls WSFO. A procedure (series of steps executed by AFOS as preprogrammed by the operator) initiated at the AFOS console, handles the entire program operation. After the data are input at the console, it is transferred to the diskette input file, and the program is run. The output file on the diskette is then stored into the AFOS data base and is called up on the console for display. The output file displayed contains the temperature ($^{o}C$) and pressure (mb) of the LCL and CCL, the convective (critical) temperature, the temperature and pressure at the top of the positive area, the energy (j/g) represented by the positive area, and the height in geopotential feet AGL of the top of the positive area. In addition, the procedure calls up on an adjacent console reported convective activity and associated positive area values from the previous year.

## 4. APPLICATIONS

The program output has many potential uses. For instance, the geopotential height of the positive area added to the stations elevation, gives a clue to the convective tops. This represents a theoretical limit to convective development, although some overshooting is likely. It should be noted that the convective temperature is, in a way, a measure of negative area (energy) or work that must be done by insolation before the parcel can reach the positive area of the sounding. If the maximum temperature forecast is less than the convective temperature, little or no convective activity should be expected.

The computation of the energy in the positive area has several possible applications. During the months of June, July, and August in 1978, this program was run on the 12Z RAOB data at the Great

| POS. AREA (j/g) | CASES | NO ACTIVITY | REPORTED CONVECTIVE ACTIVITY | | | | | HAIL SIZE (in.) | | REPORTED RAINFALL OF $\geq$ 1 in. | FUNNEL CLOUD OR TORNADO SIGHTINGS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | DSTNT CB OR OMTNS | CBS VCNTY | TRW– | TRW | TRW+ | $\leq 3/8$ | $\geq 1/2$ | | |
| 0.0 to 0.24 | 36 | 30 (83%) | 3 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.25 to 0.49 | 18 | 12 (67%) | 1 | 3 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.50 to 0.99 | 22 | 5 (23%) | 0 | 5 | 10 | 1 | 1 | 4 | 1 | 0 | 0 |
| 1.00 to 1.99 | 12 | 0 | 0 | 2 | 3 | 3 | 4 | 3 | 5 | 4 | 1 |
| 2.00 | 3 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 2 | 2 | 2 |

TABLE 1. VERIFICATION OF POSITIVE AREA WITH REPORTED CONVECTIVE EVENTS

```
GTFMCP004
WDUS00 KGTF 030142
      PRES    SFC T   850 TD  850 T    700 T    500 T    300 T    200 T
GGW [      ] [     ] [     ] [     ] [     ] [     ] [     ] [     ]
GEG [      ] [     ] [     ] [     ] [     ] [     ] [     ] [     ]
GTF [      ] [     ] [     ] [     ] [     ] [     ] [     ] [     ]
LND [      ] [     ] [     ] [     ] [     ] [     ] [     ] [     ]
BOI [      ] [     ] [     ] [     ] [     ] [     ] [     ] [     ]
SLC [      ] [     ] [     ] [     ] [     ] [     ] [     ] [     ]
DATE      MO[     ]DA[     ] YR[     ]

                                   [ ] <ENTER...THEN KEY IN C TO CONTINUE
                                        PROCEDURE.

      !!!!!!!! ENTER ALL NUMBERS WITH DECIMAL POINTS !!!!!!!!

               ALL TEMPERATURES IN C DEGREES
```

**Figure 1.** AFOS preformat for convective index program - version I program outputs positive area values (j/g) for random number of stations (up to 6), including LCL, CCL, convective temperature, and top of positive area for station #1.

```
      CONVECTIVE OUTPUT     7./ 6./1979.
      ......99.99 = CCL ABV 500 MB...POS AREA NOT MEANINGFUL.....

TEMP AT LCL=   0.9
PRESS AT LCL=  786.
TEMP AT CCL=  -2.9
PRESS AT CCL= 597.
CONVECTIVE TEMP= 30.
TEMP TOP  POS AREA= -51.4
PRES TOP POS AREA= 247.
POS AREA(J/G)=  0.74
HGT AGL TOP POS AREA=    31471.
 POSITIVE AREA AT SITE NUMBER    2 =  0.33
 POSITIVE AREA AT SITE NUMBER    3 =  0.44
 POSITIVE AREA AT SITE NUMBER    4 =  0.64
```

**Figure 2.** Example of output.

```
GTFMCP004
WOUS00 KGTF 030142
      PRES    SFC T  850 TD  850 T    700 T    500 T    300 T    200 T
GGW [     ] [     ] [     ] [     ] [      ] [      ] [      ] [      ]
GEG [     ] [     ] [     ] [     ] [      ] [      ] [      ] [      ]
GTF [     ] [     ] [     ] [     ] [      ] [      ] [      ] [      ]
LND [     ] [     ] [     ] [     ] [      ] [      ] [      ] [      ]
BOI [     ] [     ] [     ] [     ] [      ] [      ] [      ] [      ]
SLC [     ] [     ] [     ] [     ] [      ] [      ] [      ] [      ]
DATE     MO[     ]DA[     ] YR[     ]

                                [ ] <ENTER...THEN KEY IN C TO CONTINUE
                                         PROCEDURE.

       !!!!!!!! ENTER ALL NUMBERS WITH DECIMAL POINTS !!!!!!!!

                 ALL TEMPERATURES IN C DEGREES
```

**Figure 3.** AFOS preformat convective index program - version II program outputs positive area values (j/g) on map background of Pacific northwest.

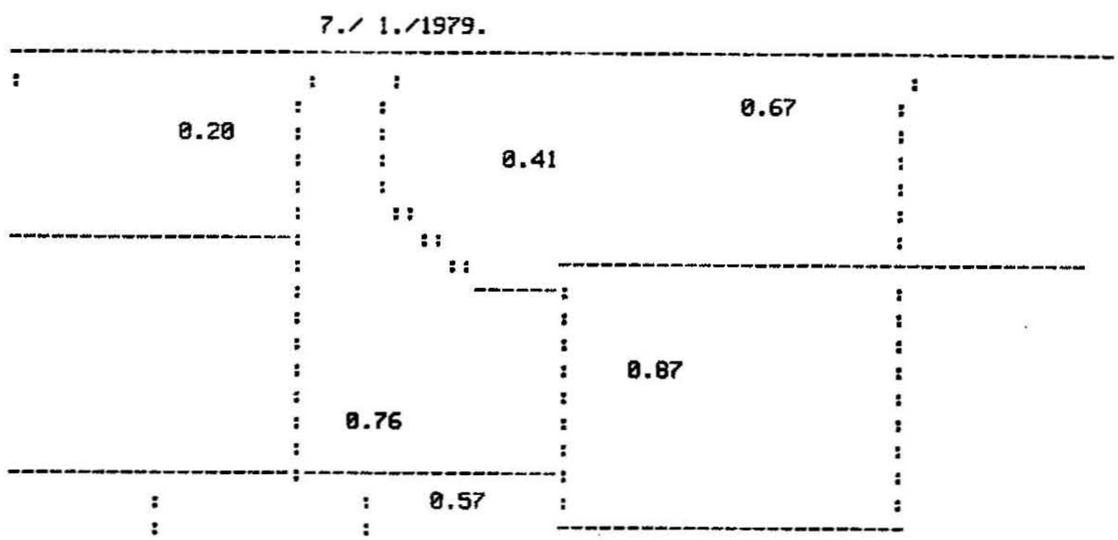



**Figure 4.** Example of output.

Falls WSFO. The positive area values (j/g) and verification are summarized in Table 1. Since this value is representative of the air mass, instead of a single fixed point, the verification data covers roughly a 100-mile radius from Great Falls. Therefore, the convective parameters in Table 1 include those reports from surrounding stations and cooperative observers within the radius. As is illustrated in Table 1, increasing values of the positive area correlate well with increasing convective activity and intensity. For example, when the positive area was equal to or greater than 1.00 j/g, hail was observed in most cases; the hail was greater than 1/2 inch in diameter in nearly half the cases. The magnitude of the positive area is also being considered as a possible parameter in a flash-flood index scheme at Great Falls.

It should be noted that the values in Table 1 were obtained from a similar computer program written for the Hewlitt-Packard 97 Calculator. This program necessarily contained more assumptions in order that the calculator could accommodate the program. The values (area) represented in Table 1 are 20 to 30 percent larger than those now obtained from the more refined FORTRAN Program given in this paper.

## 5. OPERATING SUGGESTIONS AND RESTRICTIONS

This program is quite versatile since it can be applied to most any point. Upper-air temperatures can be interpolated from the upper-air charts. It should be noted that the input of surface pressure in this program calls for uncorrected station pressure (mb), as given in the RAOB data. In addition, it is often helpful to "massage" the input variables if significant changes in the air mass are forecast.

The 850-mb dewpoint temperature input may also be determined by taking the average saturation mixing ratio in the first 100 mb - 200 mb and converting it to a 850-mb dewpoint temperature. Since small changes in the dewpoint value can produce large changes in the size of the positive area, it is imperative that a representative value of the low-level moisture is obtained.

One additional finding is that during a three-month trial period at Great Falls, it proved advantageous to run the program on upstream stations such as Spokane and Boise. Significant changes in the positive values (i.e., air mass) were often noted at these points and were considered when formulating a forecast.

Since the "parcel method" approach is used in the program, compensating downward motion in the atmosphere is neglected. This may result in a slightly exaggerated size of the positive area.

Using only 5 points to model the actual sounding (SFC, 850mb, 700mb, 500mb, 200mb) will, of course, smooth minor irregularities of the sounding. It is felt, however, that these 5 points give a sufficiently detailed structure of the sounding for the purposes of this program.

## 6. CONCLUSION

This program is run routinely at the Great Falls WSFO, using 12Z RAOB data and has been found to be a useful forecast tool. The program is currently undergoing a revision, which upon completion, will allow for multiple station entries. The output, displayed on the AFOS console, will consist of a map background of the Western Region states with the positive area values displayed near the appropriate upper-air stations. In addition to the products of this program, it also demonstrates the potential and versatility of AFOS for local applications programming. Other local studies and tasks can now be programmed and wholly integrated into AFOS, saving time and in some cases, improving results.

## ACKNOWLEDGMENTS

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