Satellite

GEOSYNCHRONOUS SATELLITE RECEIVER ORIENTATION

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ABSTRACT

To help meteorologists answer some of the general science questions they are asked, this article starts with gravitational attraction, its force at the earth's surface, which if combined with the earth's centrifugal force is called gravity, and then proceeds to equations for a circular orbit. The height of geosynchronous (stationary) satellites is determined and the geometry is shown in Figure 1. The main purpose of the paper is fulfilled in Figure 2 which provides orientation angles for geosynchronous satellite receiving antennas.

1. INTRODUCTION

Meteorologists are expected to be, and often actually are, well versed in many branches of science. One branch, frequently discussed, is astronomy. This article covers a few of the fundamentals of orbits and focuses on how to orient a receiving antenna for reception of signals from a geosynchronous (stationary) satellite.

2. SATELLITE ORBITS

Newton is given credit for the formula which gives the force of attraction, K, between two bodies of mass M and mass m separated by a distance of r: $K = GMm/r^2$ where G is the universal gravitational constant = $6.67 \times 10^{-8} cm^3 sec^{-2} gm^{-1}$. For satellites orbiting the earth, M will be the earth's mass which equals 5.959×10^{27} gm. Given the earth's radius at 40 degrees latitude = 6.367×10^8 cm, the force of gravity, g_o, acting on each gram at the earth's surface is obtained:

$$g_o = \frac{6.67 \times 10^{-8} \times 5.959 \times 10^{27} \times 1}{(6.367 \times 10^8)^2}$$

$$= 980.5 \text{ cm sec}^{-2} \text{ gm}$$

For any other distance, r, <u>from the earth's center</u> this force of attraction decreases with r^2, but still produces the centripetal force which determines the satellite orbit's radius of curvature. Since these two forces are equal: $GMm/r^2 = mv^2/r$ where v is the velocity of the satellite, with respect to space. The satellite's mass, m, cancels out of this equation. However, the energy required to get it into orbit and the energy it possesses do, of course, depend upon its mass. The equation simplifies to: $GM = v^2r$. Therefore, for any satellite in orbit v^2r is a constant. Disregarding the friction of the earth's atmosphere, if it were possible to orbit a satellite at the earth's equatorial radius, 6.378×10^8 cm, it is possible to solve for the velocity necessary to keep it in orbit:

$$v^2 = \frac{G\,M}{r} = \frac{6.67 \times 10^{-8} \times 5.959 \times 10^{27}}{6.378 \times 10^8} =$$

$$= 62.25 \times 10^{10} \text{cm}^2 \text{sec}^{-2}$$

Or, $v = 7.89 \times 10^5$ cm sec^{-1}. The period of revolution for one orbit:

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 6.378 \times 10^8}{7.89 \times 10^5}$$

$$= 5078 \text{ sec} = 1 \text{ hr } 24 \text{ min } 38 \text{ sec}$$

The velocity and period of revolution required for any other distance, r, may be easily calculated. Since $v = 2\pi r/T$, the constant v^2r can also be written: $(2\pi r/T)^2\ r = 4\pi^2 r^3/T^2$ which shows that r^3/T^2 is also a constant as stated in Kepler's third law. It is possible to take the radius and period of revolution just determined above, to calculate the distance, X, from the earth's center to a geosynchronous satellite. A "stationary" satellite must complete one circuit in 86,164 seconds, the time for the earth to complete a rotation with respect to space. It requires 24 hours (86,400 sec) to rotate with respect to the sun, but a satellite with that period of revolution would not be stationary. It follows that:

$$\frac{X^3}{(86{,}164)^2} = \frac{(6.378 \times 10^8)^3}{(5078)^2}$$

$$X = 42{,}138 \text{ km}$$

This 42,138 km minus the earth's radius, 6,378 km - 35,760 km is the height of a stationary satellite above the earth's surface. It is important to point out that the calculations are for circular orbits and that some of the input has only three significant figures, therefore only three place accuracy can be assured. An astute reader may want to try this for the most obvious earth satellite, the moon. The average distance from the earth's center to the moon's center = 384.4×10^3 km, its period of revolution = 27.32 days. The question is, does:

$$\frac{(6378\ \text{km})^3}{(5078\ \text{sec})^2} \overset{?}{=} \frac{(384.4 \times 10^3\ \text{km})^3}{(27.32 \times 86{,}400\ \text{sec})^2}$$

Well almost, but not quite; because the earth's mass is only 81 times that of the moon, they actually rotate about a common center. That center is still inside the earth, but the distance from the moon to the common center is somewhat less than 384.4×10^3 km.

3. RECEIVER ORIENTATION

Figure 1 shows the geometry involved for a geosynchronous satellite directly south of the observation site (hereafter called station). If the station is also at the equator the receiver dish should obviously be aimed vertically. The axis behind the dish is thus pointed toward Polaris. (More precisely, toward the point in the skies about which Polaris, the North Polar Star, revolves). For other latitudes the axis behind the receiver is nearly aimed at Polaris but tilted slightly southward as shown by the geometry in Figure 1. The tilt amounts to about 5 degrees at a latitude of 35 degrees. The more exact figure can be obtained from Figure 2, to be discussed shortly. The other easy determination occurs if the receiver is at the equator; then the azimuth angle is 90 degrees for any satellite east of the station. In this paper, aiming the receiver toward the east is called an azimuth of 90 degrees, toward the south, 180 degrees, etc. An Apple computer program produced the data for Figure 2 for all satellite-station locations. Although Figure 2 is labeled for northern latitudes and satellites east of the station's longitude, the geometry is symmetrical for satellites west of the station's longitude and for southern hemisphere stations. Therefore, with some relabeling, Figure 2 can be used for all satellites which turn out to be at least 2 degrees above the horizon.

As an example of the use of Figure 2, assume the station is at 42 degrees north, 88 degrees west. The satellite whose signals are to be received is over the equator at 70 degrees west. Enter the abscissa, on the bottom of the chart, with 88 minus 70 = 18 degrees difference in longitude. Also enter the ordinate, on the right side of the chart, with latitude = 42. At the intersection of those grid lines, read an elevation angle of 38 degrees and an azimuth of 154 degrees, which are the angles to be used for aiming the receiving dish. The aim should then be corrected to get the best signal. The correction should be about plus or minus one degree in either or both angles. This is necessary because the satellites wander from their assigned posts by that amount; also refraction of the signal is involved especially for those which are low, near the horizon. The accuracy of reading the graph to the nearest degree is about equal to the expected error. A second example involves a satellite at 106 west, 18 degrees west of the station's longitude. The elevation angle is the same as the first example, the azimuth would be a mirror image from 180 degrees; therefore, 180 + (180 - 154) = 206 degrees. For southern hemisphere stations the elevation angles will be no different, but the azimuth angles would have to be relabeled; the line now labeled 170 azimuth would become 10 for satellites east of the station, or 350 for satellites west of the station, etc.

4. SUMMARY

Starting with the equation for universal gravitational attraction, this paper discussed basic satellite orbits and developed an understanding of stationary satellites. The geometry of such a synchronous satellite directly south of a station is shown in Figure 1. Figure 2 displays the elevation and azimuth angles needed to aim an antenna in order to receive data from any stationary satellite. Although the azimuths shown are for northern hemisphere stations with the satellites east of the stations, symmetry allows Figure 2 to be used for satellites west of the station's longitude and for southern hemisphere stations by relabeling the azimuths.

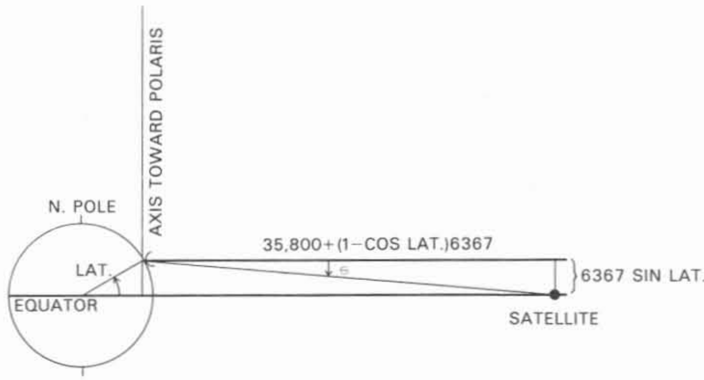

Figure 1. Geometry of a geosynchronous satellite.

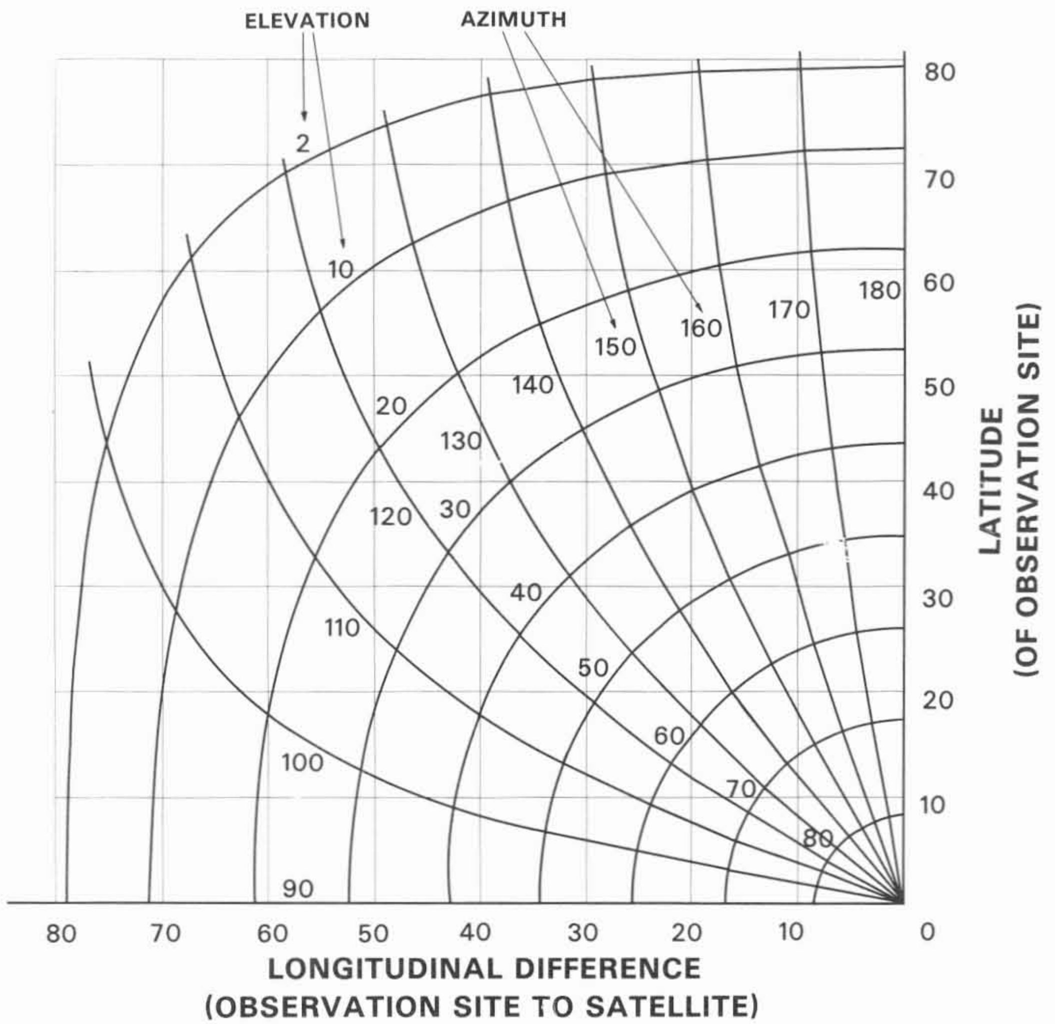

Figure 2. Receiving antenna elevation and azimuth angles.

REFERENCES AND FOOTNOTES

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