REVISING POP'S WITH CURRENT OBSERVATIONS

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ABSTRACT

A method is developed for revising the Pop in an upcoming period, based on a conditional probability argument and the Hughes and Sangster (2) formula for the joint event probability. The results follow our intuitive expectations; upward (downward) revisions for the occurrence (absence) of precipitation in the first period, with the largest revisions when the current period's forecast is poor. Nonograms are presented for two values of the Hughes-Sangster exponent.

1. INTRODUCTION

Forecasting the Probability of Precipitation (PoP) is challenging because precipitation is a discrete event in time and space which follows a non-normal frequency distribution. Although we know that precipitation events have significant autocorrelations for periods less than a day, it has not been obvious how to use that information in making quantitative weather forecasts. In particular, how does a forecaster revise the PoP partway through the forecast cycle, given the occurrence or absence of precipitation up to that time? Hovde (3) established a framework for attacking the problem but his results are counter-intuitive. The National Weather Service has been rather successful in automating the PoP forecast under the Model Output Statistics (MOS) program (4). After a numerical model has been run, the MOS equations generate an objective forecast based on statistics describing the past performance of the same model. However, while new model runs are initiated every 12 hr, forecasters must issue revised forecasts more frequently. How can a forecaster create a revised PoP, given recent observations, but an "old" MOS PoP?

Consider two consecutive forecast periods of equal length, labelled 1 and 2, representing the current period and the upcoming period (for which a revised forecast is needed). We observe that precipitation events have significant autocorrelations for periods less than a day. Thus, if precipitation has occurred in period 1, the chance is greater that it will occur in period 2. If precipitation has not occurred, the chance in period 2 is reduced. We further observe that precipitation forecasts display autocorrelations (which match the event autocorrelations for perfect forecasts). Thus, if the period 1 PoP (F1) is low, the precipitation has occurred, the period 2 PoP (F2) should be increased. High F1 with no precipitation is a signal to reduce F2. On the other hand, a high or low F1 which verifies tends to confirm the "old" F2 value. In the following discussion we quantify these comments into a simple, objective procedure for revising F1, based on "old" values of F1 and F2, and the observation of precipitation in period 1.

2. METHOD

Following the notation of Hovde (3), define three probabilities:

Ra is the probability of precipitation occurring in both periods,
Rb is the probability of precipitation occurring only in period 1,
Rc is the probability of precipitation occurring only in period 2.

Then the PoP's for these two periods are

$$F1 = Ra + Rb \quad (1a)$$

$$F2 = Ra + Rc \quad (1b)$$

Next, we repeat Hovde's observation that this problem simply describes conditional probabilities - we are seeking F2, given observations in period 1. Using the notation we have developed, the probability of P_2 (precipitation in period 2) given P_1 (precipitation in period 1) is

$$P(P_2|P_1) = \frac{Ra}{F1} \quad (2a)$$

and the probability of P_2, given n_1 (no precipitation in period 1) is

$$P(P_2|n_1) = \frac{F2 - Ra}{1-F1}. \quad (2b)$$

Hovde (3) denoted the conditional probabilities in Equations (2a) and (2b) by RIPP and NIPP, respectively.

The only unspecified variable in (2a-b) is Ra, the "joint event probability". Ra accounts for two effects, the random pairing of independent events in the two periods, and the persistence of a single event over parts of both periods. Unfortunately, no theoretical basis exists for calculating the persistence part of Ra. Hughes and Sangster (2) circumvented the problem by creating an intuitive-empirical equation:

$$Ra = L^{K} S \qquad (3)$$

where K is an empirically determined fraction, and L and S are the larger and smaller of $\{F1, F2\}$. They showed that Equation 3 satisfies the limiting cases (L=1, S=1; etc.), as well as providing the correct behavior for complete independence ($K=1$) and dependence ($K=0$). A data study of 12 hr forecasts for 8 midwestern cities resulted in values of $K=0.55$ for the cold season and $K=0.70$ in the warm season (reflecting seasonal variations in synoptic organization of precipitation). Their computation was somewhat sensitive to forecast bias. Regional variation in K was not studied, but Hughes and Sangster stated that it could be important. Also, K was not derived for 6 hr forecasts, but one should expect smaller K's due to higher autocorrelations.

3. RESULTS AND DISCUSSION

We will adopt the Hughes-Sangster form of Ra as reasonable and proceed. The revised F2 (F2') is simply the conditional probability calculated from (2a) and (2b), so for any values of F1, F2, and K, it is possible to calculate F2' for each of the cases P_1 and n_1. The formulae are so simple that even hand-held calculators can be programmed to provide F2'. Forecasters with access to PoP's in digital form could write a computer program which automatically revises F2.

As an illustration of this method's results, we have constructed nomograms for $K=0.55$ and 0.70, displayed in Figures 1 and 2 respectively. The P_1 revision is displayed in Figures 1a and 2a, while the n_1 revision is displayed in Figures 1b and 2b. To use these figures, select the correct K and observation (P_1 or n_1), find F1 on the horizontal axis, move vertically to the curve labeled with F2, and move horizontally to find the revised forecast, F2'.

The F2' results follow the qualitative arguments given in the Introduction, with monotonic changes over the whole domain. In addition, revisions to F2 are more extreme for $K=0.55$ than for $K=0.70$, consistent with greater dependence for smaller K. The dashed lines on Figures 1 and 2 are the curves along which F1=F2. The slopes of the F2 curves (solid) are discontinuous across the F1=F2 curve because the identities of L and S, used in Equation 3, are interchanged as one crosses F1=F2.

Before concluding, we will use the preceeding analysis to interpret the recent article by Hovde (3) in this journal. As referenced above, he correctly observed that F2' depends on the conditional probabilities. However, the form he assumed for Ra is too crude. As shown in the Appendix, Hovde's Ra changes its functional form at L=0.5, and this causes problems in the resulting nomograms. For example, on his Graphs 3 and 4, the 30% curves display local extrema around "prior period PoP" values of 50%. Such behavior is counter-intuitive, since one would expect the transformation to be monotonic, as it is for our results.

The work we have presented above contains several issues which deserve investigation. First, a systematic verification remains to be carried out. Furthermore, the Hughes and Sangster (2) form of Ra is a reasonable, but not rigorous expression. More fundamentally, any formula for Ra which depends on the individual PoP's alone will be incomplete. For the Hughes-Sangster Ra, we ought to make the K exponent a function of the specific synoptic situation. An approach currently under study is to derive K from the individual MOS forecast by comparing the 6 and 12 hr PoP forecasts. The most important caveat is that this method still depends on the numerical models and MOS (or whatever produces the original PoP's). The forecaster must still evaluate the basic product and decide on its usefulness for that day.

In summary, we have presented a scheme for revising second period PoP's which depends on the original PoP's for both the first and second periods, the observation of precipitation in the first period, and a choice of an empirical parameter, K. The greatest revision occurs when the first period PoP is 100% wrong, and no revision occurs when the first period PoP is exactly correct. The revision is upward (downward) when precipitation does (does not) occur in period 1. Nomograms have been provided for two values of K as an aid to the practical forecaster.

4. APPENDIX: DERIVING THE Ra IMPLIED BY HOVDE (REF. 3)

Hovde's approach to solving Equation 2 differs from that developed above, but Ra is still implicit in his work. He defines

$$Fd = Ra + Rb + Rc \qquad (A1)$$

where Fd is the probability of precipitation occurring at some time in the two periods. Also, we will define Fd* as Hovde's Fd when F1 and F2 are equal, in which case the single value of F1 and F2 is referred to as Fs:

$$Fs \equiv F1 = F2. \qquad \text{(A2)}$$

Hovde made two assumptions to specify Fd in terms of F1 and F2. First, Fd* was specified as a piecewise linear function

$$Fd^* = \begin{cases} 1.5 \times Fs & 0.0 \leq Fs \leq 0.5 \\ 0.5 \times Fs + 0.5 & 0.5 \leq Fs \leq 1.0 \end{cases} \qquad \text{(A3)}$$

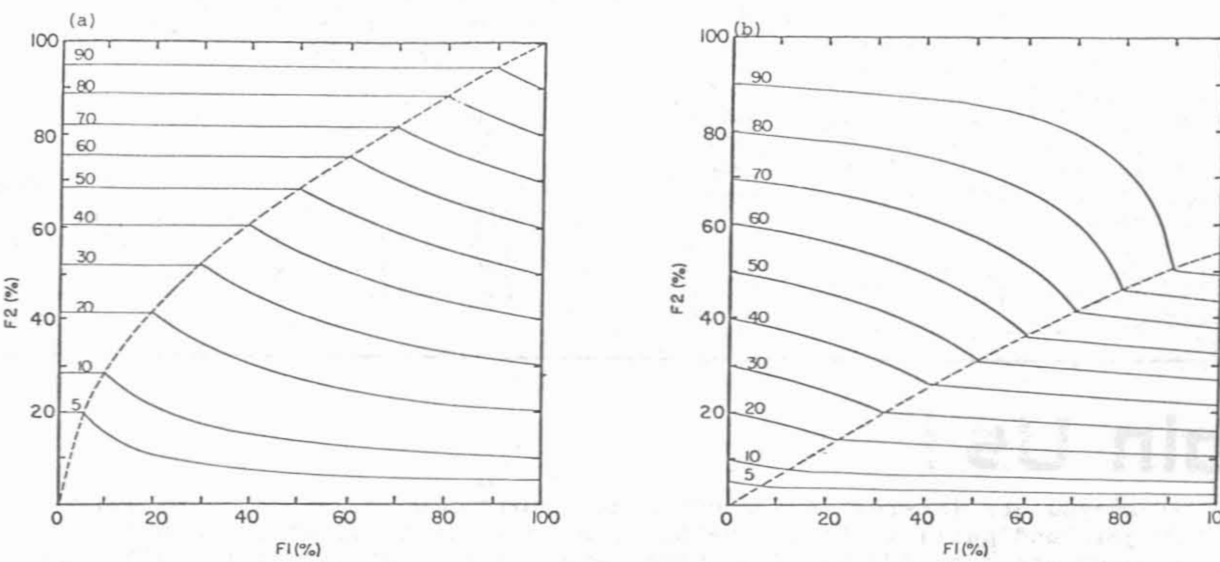
based on intuitive-empirical reasoning. Then Hovde specified Fd as a linear interpolation between L and Fd* for F1≠F2:

$$Fd = \frac{(Fd^*-L)}{L} (S + L) \qquad \text{(A4)}$$

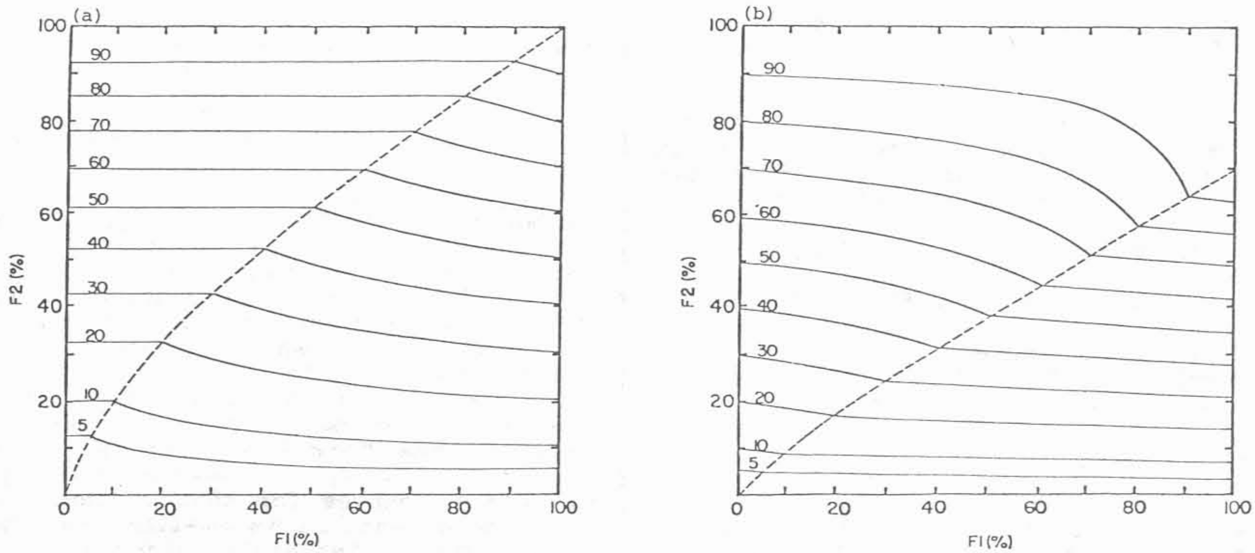
Where L and S are the larger and smaller of {F1,F2} respectively, as defined before. If we express Rb and Rc in terms of F1, F2, and Ra (from Equation 1), substitute these into (A1), apply the result to (A4), and substitute (A3) into (A4), one can show that Hovde's joint event frequency is

$$Ra = \begin{cases} \frac{1}{2} S & L<0.5 \\ \frac{3}{2} S - \frac{1}{2} \frac{S}{L} & L<0.5 \end{cases} . \qquad \text{(A5)}$$

As before, L and S are the larger and smaller of {F1, F2}. We see that the functional form of Ra changes when L is varied across the 0.5 point, while S may be varied to any value without changing the functional form of Ra. When F1 (Hovde's "prior period PoP") is larger than F2 (Hovde's "forecast period PoP"), we have L=F1, so that moving along an F2 curve varies L. Clearly, the functional change in Ra at L=0.50 is causing the reversal in slope of Hovde's F2 curves for that region. On the other hand, when F1 is smaller than F2, we have L=F2, so that moving along an F2 curve in that region varies S. There is no functional change in Ra, and consequently, those F2 curves are monotonic.



1. Nomograms for revising forecast period PoP's (F2) when the current period observation is a) P_1, and b) n_1 (respectively, precipitation has or has not occurred in period 1). Current period PoP (F1) is on the absissa, the original forecast period PoP (F2) is on the solid curves, and the revised forecast period PoP (F2') is on the ordinate. Hughes-Sangster K =0.55 is used. The original PoP's are equal along the dashed line (F1=F2).



2. Same as Figure 1, except Hughes-Sangster K =0.70.

FOOTNOTES AND REFERENCES

1. George Huffman received his B.S. degree in Physics from Ohio State University and his Ph.D. in Meteorology from M.I.T. He is Assistant Professor in the Department of Meteorology at the University of Maryland. His primary interests are synoptic-mesoscale meteorology and computer assisted realtime data problems.

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4. Lowry, D. A., and H. R. Glahn, 1976: An operational model for forecasting probability of precipitation - PEATMOS PoP. Mon. Wea. Rev., 104, 221-232.
