CALCULATOR GRADIENT WINDS

C. H. Pierce (1) discusses the approximate gradient wind equation implemented for an HP-41CV calculator by Wash & Spray (2) and suggests a preferable exact form. There, however, is a simple choice of units that leads to a universal exact form that is simpler to implement on a calculator than those mentioned by Pierce and easily includes both the ordinary and anomalous cases (Holton, 3).

Vgr

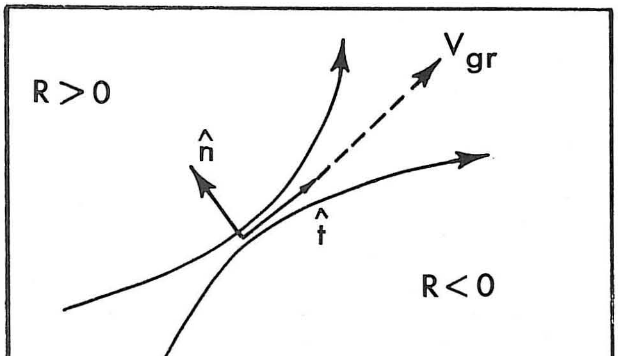

Fig. 1

In Fig. 1, take the magnitude V_{gr} of the gradient wind vector $\boldsymbol{V}_{gr}$ to be inherently positive. Introduce a tangent unit vector $\boldsymbol{t}$ parallel to $\boldsymbol{V}_{gr}$ and a righthanded normal unit vector $\hat{\boldsymbol{n}}$. For sign convention on the radius of curvature R take R>O for a path concave in the direction of $\hat{\boldsymbol{n}}$ and R<O for a path concave toward $-\hat{\boldsymbol{n}}$. This is convenient for the northern hemisphere. The southern hemisphere is most easily handled by reversing this curvature sign convention.

Measure the pressure gradient by f times the magnitude V_g of the geostrophic wind vector $\boldsymbol{V}_g$ (parallel to $\boldsymbol{V}_{gr}$). V_g, of course, is most easily calculated from geopotential slopes in pressure coordinates as on standard working charts. V_g may be either positive (V_g in the direction of V_{gr}) or negative (opposite to V_{gr}) in the anomalous cyclone case. Let (sgn R) and (sgn V_g) denote +1 or −1 corresponding to the respective signs of the radius of curvature R and geostrophic wind V_g.

The choice that makes the results utterly simple and universal is to measure V_{gr} and V_g in units of the coriolis parameter f times the radius of curvature R. I.e. twice the angular velocity of the local reference frame times R.

Let

$$V^*_{gr} \cong \frac{V_{gr}}{|fR|} \text{ and } V^*_g \equiv \frac{V_g}{|fR|}$$

The exact gradient wind equation (Holton 1979 p.62) now has the solution

$$V^*_{gr} \approx -\frac{1}{2}(\text{sgn R}) \pm \left\{\frac{1}{4} + |V^*_g| \frac{(\text{sgn } V_g)}{(\text{sgn R})}\right\}^{1/2}$$

which is a function of the single numerical value $|V^*_g|$. That is, all cases of whatever physical scale are dynamically similar and have identical numerical values of V^*_{gr} at given values of $|V^*_g|$.

The admissible cases are shown in Table 1.

Other cases are excluded by leading to negative or complex roots for real positive V^*_{gr}.

Table 1. The admissible cases.

	(sgn R)	(sgn V_g)	(sgn ( )$^{1/2}$)
ordinary cyclone	+1	+1	+1
ordinary anticyclone	−1	+1	−1
anomalous anticyclone	−1	+1	+1
anomalous cyclone	−1	−1	+1

A quite short numerical table will cover all realistic cases from synoptic scale down to mesoscale in both atmosphere and ocean. Special cases are immediately obvious such as the maximum anticyclone at $V^*_g = .25$ and $V^*_{gr} = .50$ and the inertia circle at $V^*_g = 0$ and $V^*_{gr} = 1.0$. Further, in such a table one can form the ratio $\frac{V^*_{gr}}{V^*_g} = \frac{V_{gr}}{V_g}$ which gives the ratio of gradient to geostrophic wind speed as a 1-1 function of V^*_g for each of the four cases. These values give a much clearer and more immediate impression of the relative effects in the gradient wind relation than can be obtained from the same data in physical units which are necessarily tied to the characteristic scales of each particular problem.

Table 2 gives a short selection of the results for the two ordinary cyclonic and anticyclonic cases:

For example, at $f = 10^{-4}s^{-1}$ and radius of curvature R = 1000km, fR = 100m/s and a geostrophic wind of 10m/s gives $V^{*g} = .10$ and gradient winds of (.0916)100 = 9.2m/s cyclonic or (.1127)100 = 11.3m/s anticyclonic.

Table 2. A short selection of the results for the two ordinary cyclonic and anticyclonic cases.

V^*_g	V^*_{gr} (cyclonic)	V_{gr}/V_g	V^*_{gr} (anticyclonic)	V_{gr}/V_g
.01	.0099	.9902	.0101	1.0102
.05	.0477	.9545	.0528	1.0557
.10	.0916	.9161	.1127	1.1270
.15	.1325	.8830	.1838	1.2251
.20	.1708	.8541	.2764	1.3820
.25	.2071	.8284	.5000	2.000
.30	.2416	.8054	—	—
.35	.2746	.7846	—	—

NOTES AND REFERENCES

1. Pierce C. H. 1986: Letter to the editor. Nat Wea Dig 11, 2, p. 3.

2. Wash, C. H. and L. A. Spray, 1985: Weather Analysis Programs Using The HP-41CV, Nat. Wea. Dig. 10, 4, pp. 20–29.

3. Holton J. R. 1979: An Introduction to Dynamic Meteorology. *Academic Press pp. 56–64.*

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