A PROGRAM FOR CALCULATING THE MONTGOMERY STREAM FUNCTION

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Abstract

Many current upper-air analysis packages perform isentropic analysis. However, some of those packages do not calculate the Montgomery stream function (M) on isentropic surfaces. This may make it difficult to accurately determine ageostrophic forcing from differences between geostrophic and total winds since geostrophic winds are proportional to gradients of M on isentropic surfaces. Starting from theory, this paper derives an approach for finding M from isobaric data. The approach is then translated into a FORTRAN program which computes M. For illustrative purposes, the program processes user-supplied upper-air data. Two data sets are provided to help the user test resulting code. Meteorologists could adapt the code for inclusion in upper-air or gridded-data analysis packages which do not currently calculate M. The author has regularly used the code to produce isentropic analyses since 1993.

1. Introduction

There are a number of programs which analyze upper-air observations and gridded model output. Many of these programs include some level of isentropic analysis. However, some of those programs (e.g., PCGRIDDS) (NWS 1994) do not calculate the Montgomery stream function $(M = C_P T_\theta + g z_\theta)$ (Montgomery 1937) on isentropic surfaces. M is crucial to isentropic analysis since geostrophic wind speeds on isentropic surfaces are proportional to gradients of M. (M performs a role on isentropic surfaces which is analogous to geopotential height on isobaric surfaces.) Some current isentropic analysis packages (e.g., PCGRIDDS) determine geostrophic winds on isentropic surfaces by interpolating them from isobaric surfaces. While this may outwardly appear to be a reasonable procedure, it does not guarantee that the resulting geostrophic winds are as accurate, or as representative, as they could be. Problems with geostrophic winds affect ageostrophic winds derived from vector differences between geostrophic and total winds.

Starting from theory, this paper derives an approach for finding *M* from isobaric data. The approach is translated into a FORTRAN program which computes *M*. For illustrative purposes, the program is set up to process user-supplied upper-air data. Two data sets are provided to help the user test resulting code. In principle, the program could be adapted for use in upper-air or gridded-data analysis packages which do not calculate *M*. Meteorologists familiar with macros may be able to translate the program into a macro *provided that the analysis package of interest allows the user to find the nearest isobaric surfaces above and below an isentropic surface of interest.* Not all analysis packages allow this. The paper proceeds as follows. The mathematical basis of the program is developed in section

2. The results of the program from two data sets are given in section 3. Section 4 summarizes the results. The program and data sets are given in the Appendix.

2. Mathematical Basis of the Program

In the 1930s, upper-air analyses were performed on constantaltitude, isobaric and isentropic (θ) surfaces. At that time the merits of each type of chart were actively debated. Debate continued and by the end of World War II, isobaric charts were routinely analyzed by forecasters in the U.S. Weather Bureau (WB) (Fulks 1945). Isentropic analysis was not regularly performed again until the 1950s and 1960s.

One possible reason for the temporary demise of isentropic analysis was inaccurate Montgomery stream functions which resulted in geostrophic wind laws which did not appear to work on isentropic charts (Bleck 1973; Moore 1988). In retrospect, Brooks (1942) almost found the reason for those incorrect computations. He observed that C_PT_θ and gz_θ varied inversely and individually much faster than their sum $M = C_P T_\theta + g z_\theta$, so that in terms of numerical accuracy, it was desirable to calculate a small quantity M' defined as $M' = M - C_P \theta$ rather than M. However, it was Danielsen (1959) who discovered the actual reason for the incorrect computations. He noted that the WB had incorrectly calculated M. They separately interpolated z_{θ} and T_{θ} in the two terms which comprise M and thereby unwittingly violated hydrostatic consistency constraints imposed on each term. Using Danielsen's work, Mahlman and Kamm (1965) (henceforth, MK65) and later Reiter (1972) proposed and tested a method of calculating M whose accuracy rivaled standard methods of determining geopotential height along isobaric surfaces. This method will be the basis for the program in the Appendix. MK65 started with the total differential of M

$$dM = \frac{\partial M}{\partial T_{\theta}} dT_{\theta} + \frac{\partial M}{\partial z_{\theta}} dz_{\theta} + O(dT_{\theta}^{2} + dz_{\theta}^{2})$$
 (1)

which becomes

$$dM = C_P dT_\theta + g dz_\theta \tag{2}$$

after substituting the definition of M for the partial derivatives in (1) and neglecting higher-order terms. An expression for dz_0 is derived from the hypsometric equation,

$$z_{\theta} = z_{e} + \frac{R\overline{T}}{g} \ln \left(\frac{P_{e}}{P_{\theta}} \right)$$
 (3)

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whose total differential is

$$dz_{\theta} = dz_{e} + \frac{R}{g} \ln \left(\frac{P_{e}}{P_{\theta}} \right) d\overline{T} + \frac{R\overline{T}}{g} \frac{dP_{e}}{P_{e}}$$

$$- \frac{R\overline{T}}{g} \frac{dP_{\theta}}{P_{\theta}} + HOT$$
(4)

Danielsen substituted (4) into (2) and neglected higher-order terms (HOT) to produce

$$dM = gdz_{e} + R \ln \left(\frac{P_{e}}{P_{\theta}}\right) d\overline{T} + R\overline{T} \frac{dP_{e}}{P_{e}}$$

$$- R\overline{T} \frac{dP_{\theta}}{P_{\theta}} + C_{P}dT_{\theta}.$$
(5)

He assumed that the differentials represent errors so that

$$\Delta M = g\Delta z_e + R \ln \left(\frac{P_e}{P_\theta}\right) \Delta \overline{T} + \frac{R\overline{T}}{P_e} \Delta P_e$$

$$+ \left[-R\overline{T} \frac{\Delta P_0}{P_\theta} + C_P \Delta T_\theta \right]$$
(6)

where ΔM is the error made if height, pressure and temperature of an isentropic surface $(Z_{\theta}, P_{\theta}, T_{\theta})$ and T are determined independently of each other.

Danielsen noted that an expression analogous to (6) for geopotential height along an isobaric surface is derived from the total differential of Φ_n :

$$\Delta \Phi_{P} = g \Delta z_{e} + R \ln \left(\frac{P_{e}}{P_{p}} \right) \Delta \overline{T} + \frac{R \overline{T}}{P_{e}} \Delta P_{e}. \tag{7}$$

Comparing (6) and (7), it is obvious that bracketed terms in (6) constitute an additional error that arises when P_{θ} and T_{θ} are considered to be independent. However, Danielsen noted that P_{θ} and T_{θ} are not independent, since they are related by Poisson's equation. Therefore, errors contributed by the bracketed term in (6) may be reduced by relating ΔP_{θ} to ΔT_{θ} with a differentiated form of Poisson's equation. This is accomplished by first logarithmically differentiating Poisson's equation on an isentropic surface

$$\frac{1}{T_{\theta}}dT_{\theta} - \frac{R}{C_{\theta}P_{\theta}}dP_{\theta} = \frac{1}{\theta}d\theta = 0$$
 (8)

then rearranging the result to produce a relation for dT_{θ}

$$C_P dT_\theta = \frac{RT_\theta}{P_\theta} dP_\theta \tag{9}$$

and then substituting this relation into (5) to produce a new expression for dM

$$dM = gdz_{e} + R\ln\left(\frac{P_{e}}{P_{\theta}}\right)d\overline{T} + \frac{R\overline{T}}{P_{e}}dP_{e} + (10)$$

$$[(T_{\theta} - \overline{T})R\frac{dP_{\theta}}{P_{e}}]$$

so that a new expression for the error ΔM becomes

$$\Delta M = g\Delta z_e + R \ln \left(\frac{P_e}{P_\theta}\right) \Delta \overline{T} + \frac{RT}{P_e} \Delta P_e + (11)$$
$$[(T_\theta - \overline{T}) R \frac{\Delta P_\theta}{P_\theta}].$$

Note that the bracketed error term in (6) has been rewritten as

$$[(T_{\theta} - \overline{T}) R \frac{\Delta P_{\theta}}{P_{\alpha}}]. \tag{12}$$

so that the errors contributed to M by the bracketed term in (11) now depend only on pressure errors (ΔP_{θ}) instead of pressure and temperature errors (ΔP_{θ} and ΔT_{θ}).

Errors contributed by (12) can be made vanishingly small near the earth's surface (i.e., large values of P_{θ}), but even in the mid-troposphere, the errors are insignificant. Danielsen (1959) noted that an error of 2 mb at 500 mb ($\Delta P_{\theta} = 2$ mb, $P_{\theta} = \underline{5}00$ mb), together with a 10 K temperature difference ($T_{\theta} - T$) in (12) contributes to a stream function error of only 12 m² s⁻², which approximately corresponds to an error of 1.2 m in the height of an isobaric surface. This is much less than errors normally associated with radiosonde isobaric height measurements. On the other hand, an independent error of $\Delta T_{\theta} = 0.2$ K combined with the same 2-mb error ΔP_{θ} in the rightmost term of (6) produces an error in M of 600 m² s⁻², which is significant because it represents an error of approximately 60 m in the height of an isobaric surface. Therefore, the computational procedure illustrated by (10) can produce M computations along isentropic surfaces whose accuracies rival geopotential along isobaric surfaces.

A working equation and a procedure for calculating M from upper-air observations proceeds from Danielsen's work and MK65. M in terms of the height of an isentropic surface z_{θ} and the isobaric surface below the isentropic surface z_{b} is written as

$$M = C_P T_{\theta} + g z_h + g (z_{\theta} - z_h). \tag{13}$$

The rightmost term in (13) is found from the hypsometric equation

$$z_{\theta} - z_{b} = -\frac{R\overline{T}}{g} \ln \left(\frac{P_{\theta}}{P_{b}} \right)$$
 (14)

so that (13) becomes

$$M = C_P T_\theta + g z_b + R \overline{T} \ln \left(\frac{P_b}{P_\theta} \right). \tag{15}$$

As noted by Danielsen, P_{θ} and T_{θ} are not independent, but are related by Poisson's equation. Poisson's equation is solved for P_{θ} to obtain

$$P_{\theta} = 1000 \left(\frac{T_{\theta}}{\theta} \right)^{CP/R} \tag{16}$$

which is then substituted into (15) to produce

$$M = C_P T_\theta + g z_b + R \overline{T} \ln \left(\frac{P_b}{1000} \left(\frac{\theta}{T_\theta} \right)^{C_{P/R}} \right)$$
 (17)

which is the working equation for M.

Equation (17) is evaluated with the following procedure:

- (1) Specify the isentropic surface of interest θ .
- (2) Find the temperature T_{θ} of the isentropic surface θ by linearly interpolating temperatures of the nearest isobaric levels above (subscript a) and below (subscript b) the isentropic surface θ

$$T_{\theta} = T_b + (T_a - T_b) \left(\frac{\theta - \theta_b}{\theta - \theta_b} \right). \tag{18}$$

(3) Assign z_b and P_b to the height and pressure of an isobaric level below P_{θ} .

(4) Determine \overline{T} from the average of T_{θ} and T_{b}

$$\overline{T} = \frac{T_b + T_\theta}{2} \tag{19}$$

(5) Substitute results from steps (1)–(4) into (17) to find M.

These steps can be translated into common computer languages, or encoded as a macro in upper-air and gridded-data processing packages which allow the user to find the nearest isobaric surfaces above and below an isentropic surface of interest. A simple FORTRAN program which computes M is shown in the Appendix. If the comments are removed, the program is very short and easily typed with a text editor or word processor. The program assumes that the user has information from isobaric surfaces above and below the isentropic surface of interest. In practice this information would likely be contained in arrays so that the user would reference array elements instead of READ statements (or other language-specific input statements). Input isobaric data and M for two analyses along the 319.5 K isentropic surface are given in Tables 1 and 2 in the Appendix. The data are taken from 3-h CLASS radiosonde observations at 2100 UTC 8 March 1992 - 0000 UTC 9 March 1992 during STORM-FEST (STormscale Operational and Research Meteorology-Fronts Experiment Systems Test) (Cunning and Williams 1993). As part of STORM-FEST, flow along the 319.5 K isentropic surface was followed by an instrumented aircraft (Prater 1994) during lee cyclogenesis (Mahoney et al. 1995).

3. Results

Figure 1 shows M (thick lines), geostrophic wind isotachs (thin, dashed lines), pressure (thin, solid lines) and observed winds on the 319.5 K isentropic surface at 2100 UTC 8 March 1992. M was computed using the first data set in the Appendix. M was contoured at a 1200 m² s⁻² interval to aid comparison with height on isobaric charts contoured at a 120 m interval. The figure shows a trough over the southern Rockies and the exit region of a jet. Observed wind speeds over southern New

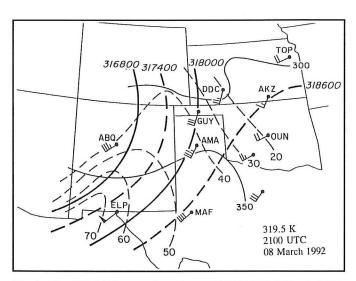


Fig. 1. The 319.5 K isentropic surface at 2100 UTC 8 March 1992. M computed from the program (thick lines and $\rm m^2\,s^{-2}$); isobars (thin lines and mb); observed winds (m s⁻¹); full barb = 10 m s⁻¹; flag = 50 m s⁻¹; geostrophic wind isotachs (dashed lines and m s⁻¹).

Mexico and west Texas are sub-geostrophic, as expected in cyclonically-curved flow. In the operational environment, the quality of an isentropic analysis is often assessed by comparing it with a nearby isobaric surface. That approach will be used here. Height, geostrophic wind isotachs and observed winds on the nearby 320-mb isobaric surface are shown for comparison in Fig. 2. Figures 1 and 2 show reasonable qualitative agreement between the pressure gradient fields and geostrophic wind speeds, with exception of southwest Texas where the 319.5 K isentropic surface was closer to 400 mb. Another example of *M* on the 319.5 K isentropic surface from the program at 0000 UTC 9 March 1992 (the second data set in the Appendix) is shown in Fig. 3. This analysis shows good time continuity with Fig. 1, with eastward motion of the trough and northeastward propagation of the jet streak. Analysis of the nearby 320-mb

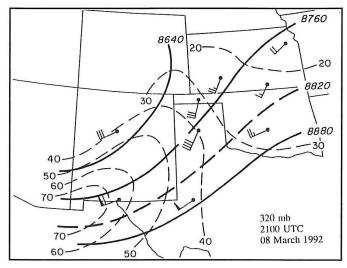


Fig. 2. 320-mb isobaric analysis at 2100 UTC 8 March. Height (thick lines and meters); observed winds (m s⁻¹) (same barb convention as in Fig. 1); geostrophic wind isotachs (dashed lines and m s⁻¹)

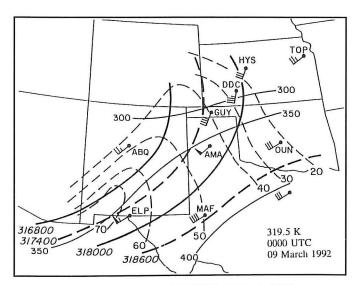


Fig. 3. Same as Fig. 1 except 0000 UTC 9 March 1992.

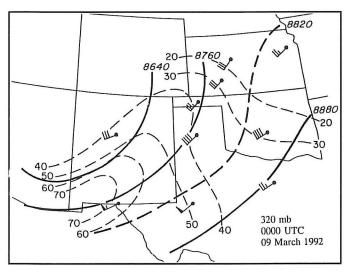


Fig. 4. Same as Fig. 2 except 0000 UTC 9 March 1992.

isobaric surface is shown in Fig. 4 for comparison. Figures 3 and 4 show reasonable qualitative agreement with exception of eastern Oklahoma and Texas where the 319.5 K isentropic surface was closer to the 400-mb level. Although only two isentropic analyses are presented, both show reasonable consistency with analyses on nearby isobaric surfaces. Other examples of M computed using a similar approach and isobaric surfaces are shown by Reiter (1972). The author has used the program in the Appendix since 1993 to regularly generate M using an upper-air analyses package adapted from MK65.

4. Summary

There are a number of programs which analyze upper-air observations and gridded model output. Many programs include some level of isentropic analysis. However, some of those programs (e.g., PCGRIDDS) do not analyze the Montgomery stream function (M) on isentropic surfaces. Starting from theory, this paper presented an approach for finding M from isobaric data. The approach was translated into a FORTRAN program which computes M. For illustrative purposes, the program is set up to process user-supplied upper-air data. Two isentropic analyses featuring M from the program were shown. Those analyses showed good time continuity and reasonable qualitative agreement with features on nearby isobaric surfaces. The author has used the program successfully since 1993 in an upper-air analyses package adapted from MK65. Meteorologists are encouraged to test and refine the program, and implement the code in PCGRIDDS and other upper-air analysis packages which do not calculate M.

Acknowledgments

The author wishes to thank Drs. John D. Marwitz and Thomas R. Parish of the University of Wyoming (UW) for their assistance. This work was derived from a very small part of the author's dissertation at UW. The author also thanks Ms. Susan Allen at UW for drafting the figures. The author is indebted to Dr. James T. Moore of St. Louis University and one other (anonymous) reviewer for their comments. Dr. Moore also teaches a course on isentropic analysis at the National Weather

Service Training Center in Kansas City which the author had the pleasure of attending several years ago. The author's research at UW was supported by National Science Foundation grant ATM-9308409.

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APPENDIX: PROGRAM AND DATA			C***	C*****************		
_	ONT.FOR HIS CODE ILLUSTRATES A PROGRAM FOR COM- UTING THE MONTGOMERY STREAM FUNCTION. RITTEN BY ERWIN T. PRATER AUGUST, 1993 FOR PPER-AIR ANALYSIS		С	PROGRAM MONT		
C C TH				REAL SURF,TTHETA,TA,TB,TBAR,M,R,CP CHARACTER*3 STAT INTEGER IMAX		
C WI C UP C			C	GAS CONSTANTS (MKS UNITS) R = 287.04 CP = 1004.6		
C WRITTEN AND COMPILED ON AN IBM-COMPATI- C BLE 486DX2 USING MICROSOFT FORTRAN 77. CODE C BASED ON MAHLMAN AND KAMM (65). C			С	GRAVITATIONAL CONSTANT (MKS UNITS) G=9.806		
C****	********************		C	SPECIFY THE ISENTROPIC SURFACE OF		
	ARIABLES ENTERED BY THE USER:		C C	INTEREST AND COLLECT INFORMATION ABOUT NEARBY ISOBARIC SURFACES.		
C	STAT	= THREE-LETTER STATION ID		WRITE(*,*) 'ENTER THE TEMPERATURE (K) OF		
C C C	SURF	= ISENTROPIC SURFACE OF INTEREST (KELVIN)		THE THETA SURFACE' READ(*,*) SURF		
C C C C	TA	= TEMPERATURE OF THE NEAREST ISOBARIC SURFACE ABOVE THE ISENTROPIC SURFACE (CELSIUS)	C C C	IMAX SPECIFIES THE NUMBER OF LOCATIONS WHERE THE MONTGOMERY STREAM FUNCTION WILL BE CALCULATED. IMAX IS SET TO 9 IN THIS PROGRAM. IN PRACTICE IT HAS NO		
00000000000	ТВ	= TEMPERATURE OF THE NEAREST ISOBARIC SURFACE BELOW THE ISENTROPIC SURFACE (CELSIUS)	С	PRACTICAL LIMIT. IMAX = 9		
	PA	= PRESSURE OF THE NEAREST ISO- BARIC SURFACE ABOVE THE ISEN- TROPIC SURFACE (MILLIBARS)		DO 2 I = 1,IMAX WRITE(*,*) 'ENTER THE THREE-LETTER STA- TION IDENTIFIER'		
	PB	= PRESSURE OF THE NEAREST ISO- BARIC SURFACE BELOW THE ISEN- TROPIC SURFACE (MILLIBARS)	1	READ(*,1) STAT FORMAT (A3) WRITE(*,*) 'ENTER THE TEMPERATURE (C)		
C C C	ZB	= GEOMETRIC ALTITUDE OF AN ISO- BARIC SURFACE BELOW THE ISEN- TROPIC SURFACE (METERS)		AND PRESSURE' WRITE(*,*) '(MB) OF THE CLOSEST ISOBARIC SURFACE ABOVE' WRITE(*,*) 'THE ISENTROPIC SURFACE OF		
C C INTERNALLY-COMPUTED VARIABLES:				INTEREST' READ(*,*) TA,PA		
C C C	IMAX	= NUMBER OF LOCATIONS WHERE M IS COMPUTED		TA = TA + 273.15 WRITE(*,*) 'ENTER HEIGHT, TEMP. (C) AND		
000000000000	TTHETA = TEMPERATURE OF THE ISENTROPIC SURFACE THETAA = POTENTIAL TEMPERATURE OF THE ISOBARIC SURFACE ABOVE THE ISENTROPIC SURFACE OF INTEREST			PRESSURE' WRITE(*,*) '(MB) OF THE CLOSEST ISOBARIC SURFACE BELOW' WRITE(*,*) 'THE ISENTROPIC SURFACE OF INTEREST' READ(*,*) ZB,TB,PB TB = TB + 273.15		
	THETA	B = POTENTIAL TEMPERATURE OF THE ISOBARIC SURFACE BELOW THE ISENTROPIC SURFACE OF INTEREST	C C	COMPUTE THE TEMPERATURE OF THE ISENTROPIC SURFACE OF INTEREST (TTHETA)		
	TBAR	= MEAN TEMPERATURE BETWEEN PB AND THE ISENTROPIC SURFACE (KELVIN)		THETAA = $TA*((1000./PA)**(R/CP))$ THETAB = $TB*((1000./PB)**(R/CP))$ TTHETA = $TB + (TA-TB)*((SURF-THE TAB)/(THETAA-THETAB))$		
C C C C	M	= MONTGOMERY STREAM FUNCTION (m**2/s**2)	C C	COMPUTE THE MEAN TEMPERATURE (TBAR) BETWEEN THE ISENTROPIC SURFACE OF		

C INTEREST AND THE ISOBARIC SURFACE BELOW

TBAR = (TTHETA + TB)/2.

C COMPUTE M

M = CP*TTHETA + G*ZB + R*TBAR* & ALOG((PB/1000.)*((SURF/TTHETA)**(CP/R)))

C WRITE THE COMPUTED M AND STATION ID

WRITE(*,*) 'STATION ', 'MONT. SFN. (m**2/s**2): '

WRITE(*,3) STAT,M

3 FORMAT (A4,8X,F8.0)

2 CONTINUE

STOP END

LIST OF SYMBOLS

- a, b subscripts for variables above and below a specific level, respectively
- C_P specific heat of dry air at constant pressure (1004.6 J kg⁻¹ K⁻¹)
- e subscript denoting the Earth's surface
- g gravitational constant (9.806 m s⁻²)
- M Montgomery stream function $(M = C_P T_\theta + g z_\theta)$
- M' modified Montgomery stream function
- P pressure; subscript for an isobaric surface
- Φ geopotential height
- R dry gas constant (287.04 J kg $^{-1}$ K $^{-1}$)
- T absolute temperature
- \overline{T} mean absolute temperature
- θ potential temperature; subscript for an isentropic surface
- z geometric altitude

Table 1. Input data and M 2100 UTC 8 March 1992. M is contoured in Fig. 1.

Station and three-letter identifier	Temperature (°C) and pressure (mb) above $\theta = 319.5$ K	Height (m), temperature and pressure below $\theta = 319.5 K$	<i>M</i> (m² s⁻²)
Topeka, KS (TOP)	-50.7 280	9493 -49.0 290	318438
Arkansas City, KS (AKZ)	-38.0340	8244 - 36.4 350	318630
Dodge City, KS (DDC)	-48.7290	9236 - 47.4 300	318108
Guymon, OK (GUY)	-44.0 310	8796 -42.2 320	318031
Albuquerque, NM (ABQ)	-38.9 330	8212 - 39.7 340	316352
Amarillo, TX (AMA)	-37.3 340	8199 - 36.7 350	318189
Norman, OK (OUN)	-38.1 340	8255 -36.6 350	318739
El Paso, TX (ELP)	$-32.4\ 370$	7588 -30.9 380	317851
Midland, TX (MAF)	-33.4 360	7867 -32.6 370	318739

Table 2. Input data and M 0000 UTC 9 March 1992. M is contoured in Fig. 3.

Station and three-letter identifier	Temperature (°C) and pressure (mb) above θ = 319.5K	Height (m), temperature and pressure below $\theta = 319.5$ K	<i>M</i> (m² s⁻²)
Topeka, KS (TOP)	-40.2 330	8426 - 38.6 340	318453
Hays, KS (HYS)	-48.5 290	9228 -46.9 300	318033
Dodge City, KS (DDC)	-53.0270	9647 - 51.2 280	317700
Guymon, OK (GUY)	-40.0 330	8342 - 38.4 340	317630
Albuquerque, NM (ABQ)	-42.4 310	8599 - 43.0 320	316100
Amarillo, TX (AMA)	-34.3360	7760 -33.0 370	317689
Norman, OK (OUN)	-34.0 360	7838 - 32.7 370	318455
El Paso, TX (ELP)	-38.1 340	8159 - 36.6 350	317796
Midland, TX (MAF)	-32.2 370	7660 - 30.9 380	318557