A SIMPLE TURBULENT KINETIC ENERGY EQUATION AND AIRCRAFT BOUNDARY LAYER TURBULENCE

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Abstract

The basic science of turbulence has been known for many years, yet the knowledge has hardly become a part of operational aviation turbulence forecasting. The history of turbulence forecasting, which has primarily concentrated at elevations above the boundary layer, has been an amalgamation of empirical rules of thumb and forecaster intuition, not of strong theory and conceptual models. Turbulence sources are examined from the turbulent kinetic energy equation. Data from a year-long study at the NOAA/National Weather Service/Aviation Weather Center shows that even a simple first order closure scheme has significant diagnostic skill in the boundary layer for aviation purposes.

1. Introduction

The flow of energy in the atmosphere cascades from the larger scales to the smaller. Depending on the reference scale, eddy motions on smaller scales are considered turbulent. Eventually energy is dissipated on the molecular scale. The equations of motion describe these processes, but as every meteorologist has learned, the equations cannot be closed, and there are leftover terms that cannot be resolved completely. These leftover terms describe, or more precisely account for, the turbulence.

The most common approach to account for turbulence is to estimate its energy. Turbulence kinetic energy (TKE) equations have traditionally been an important part of atmospheric numerical models because models that do not account for the dissipation of energy at scales smaller than the grid spacing will eventually become unstable. TKE equations range from the simple first order equations to much more complex second, third, and fourth order equations (Mellor and Yamada 1982).

Turbulence affects aircraft in flight by causing fluctuations in altitude and speed. The turbulence may be strong enough to cause pilots to lose control. At cruise altitudes, the pilot usually has enough time to regain control. Control loss near the ground can result in a crash. Since all aircraft are in the atmospheric boundary layer[1] sometime during their flight, boundary layer turbulence is a concern for all who fly.

Because of the potential hazard, meteorologists at the Aviation Weather Center (AWC), Kansas City, Missouri, and other centers around the world diagnose and forecast turbulence at all levels which aircraft fly, including the boundary layer. To date, they have relied on an amalgamation of mostly empirical rules and equations mostly applied above the boundary layer (Dutton 1980, and Knox 1997). Some of these empirical rules are based on connections between observed atmospheric patterns and aircraft turbulence reports while others can be traced back to more fundamental processes described in a typical TKE equation. Diagnostic and forecast methods for aviation turbulence in the boundary layer are practically non-existent, although a few, mostly unpublished, studies exist. Amburn (1992) subjectively related the occurrences of strong winds, solar insolation, and low-level atmospheric lapse rates to reports of aircraft turbulence. Even a simple TKE equation describes how these atmospheric characteristics produce turbulence.

Aviation meteorologists should become familiar with TKE equations. They were primarily developed from understandings of boundary layer turbulence and should apply directly to their forecast problem. After introducing some background concepts and simplifying the general TKE equation, this paper shows that output from the simple TKE equation is related to aircraft-reported turbulence in the boundary layer.

2. Background

The general TKE equation follows from the equations of motion and continuity. Garratt (1992) is just one of numerous textbooks that show its derivation. The procedure is to break the wind into the sum of its mean component and the fluctuating component ($v = \bar{v} + v'$), substitute into the equation of motion, then multiply the equation by its fluctuating component (v'). Defining $e = |v'|^2/2$ as the kinetic energy of the fluctuations (TKE), making reasonable assumptions and ignoring small terms, the resulting equation becomes

$$\frac{\partial e}{\partial t} = -\langle v'w' \rangle \frac{\partial \bar{v}}{\partial z} + \frac{g}{\bar{\Theta}} \langle w'\Theta' \rangle - \frac{\partial \langle w'e \rangle}{\partial z} - \frac{\partial (\langle w'p' \rangle / \rho_0)}{\partial z} - \varepsilon \quad (1)$$

where v is a characteristic horizontal wind velocity, w is the vertical wind, g is the gravitational acceleration, Θ is the potential temperature, p is the pressure, ρ_o is the density, and ε is the molecular dissipation. The angle braces indicate the average over space of the perturbed quanti-

[1] The atmospheric boundary layer is the layer of air in which the effects of the Earth's surface are felt directly (Garratt 1992). It is usually the lowest kilometer above ground level.

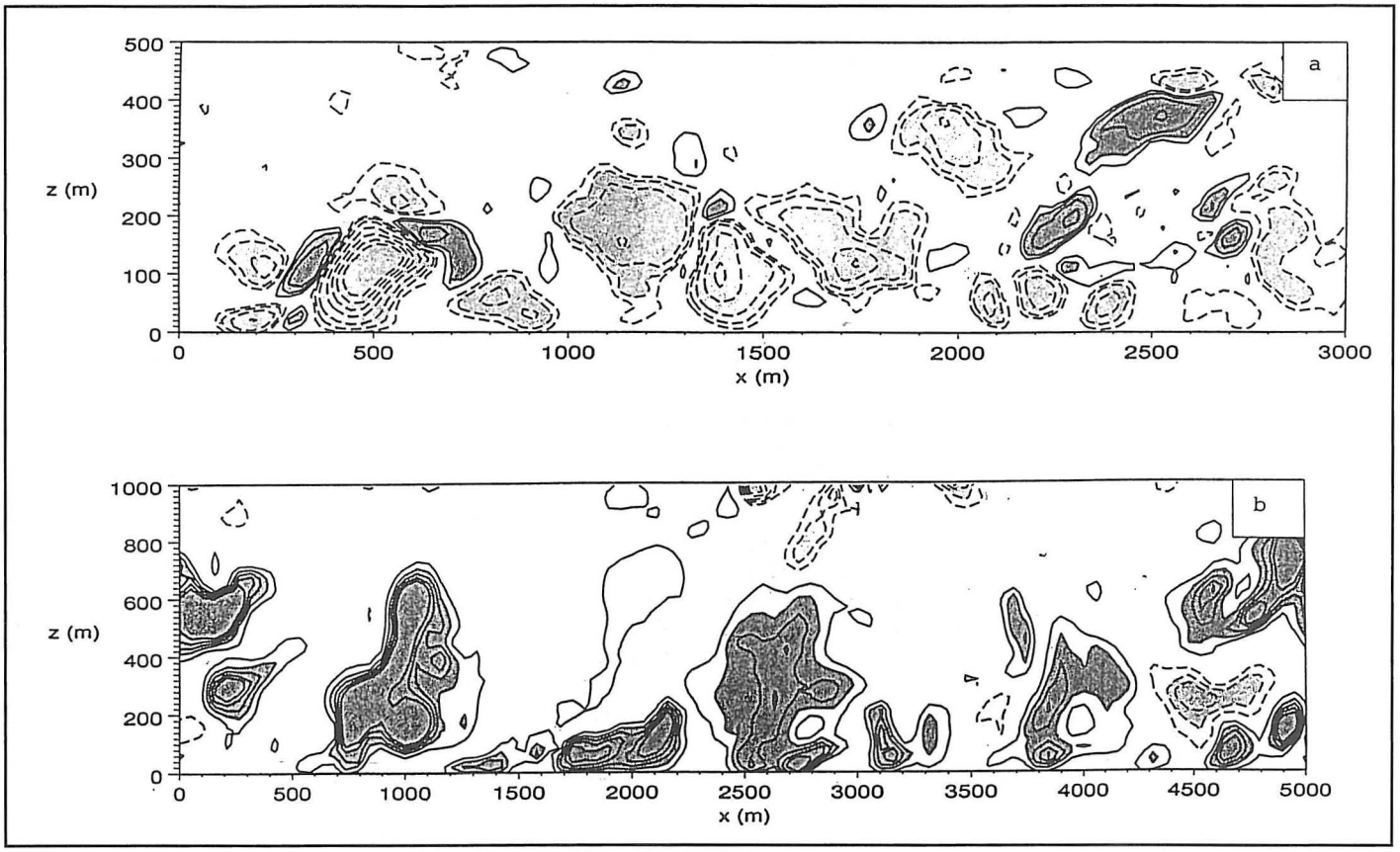


Fig. 1. From Moeng and Sullivan's (1994) large eddy simulations, snapshot cross sections of contours of eddy correlations leading to turbulence production a) in the shear-dominated simulation and b)in the buoyancy-dominated simulation. In a) the lightly shaded values are significantly negative indicating zones of positive TKE production. In b) the darkly shaded values are significantly positive again indicating zones of positive TKE production. Note the change in vertical scale between a) and b).

ties, those that are primed. The first two terms on the right of (1) are the TKE production due to wind shear (S) and the TKE production due to buoyancy (B). Note that it is the wind and temperature perturbations in the vertical that produce the TKE. Terms 3 and 4 move TKE vertically by turbulence (T) and pressure perturbations (P) and are usually combined into one transport term (T + P). Terms 1 and 2 are TKE sources, while term 5 is the TKE sink. The molecular dissipation rate, ε, is proportional to $e^{3/2}$ (Moeng and Sullivan 1994).

Moeng and Sullivan (1994) describe the importance of each term in the TKE equation in various flow states of the boundary layer using a large-eddy simulation (LES). An LES is a very high resolution numerical model that explicitly solves the primitive equations for the large turbulent eddies. Resolutions are on the order of tens of meters.

When shear dominates the flow, most of the shear is concentrated in the lowest portions of the boundary layer, so most of the eddies and the TKE production are located there. Figure 1a shows a cross section snapshot of the $\langle v'w' \rangle$ in their shear-dominated LES. The lighter shaded areas are of negative $\langle v'w' \rangle$, and from (1) are regions of positive TKE production. The mean budget in Fig. 2a shows that shear production nearly balances dissipation in this LES. There is very little contribution to the TKE from B or (T + P).

When buoyancy dominates the flow, the eddies fill the boundary layer instead of being concentrated near the ground. Figure 1b shows similar cross section snapshot of $\langle w'\Theta' \rangle$. There is more buoyancy TKE production at higher altitudes than similar TKE production in the shear case. In fact, the TKE budget in Fig. 2b shows that buoyancy TKE production decreases more slowly with height than shear TKE production. Furthermore, transport of the TKE, (T + P), into the upper portion of the boundary layer is significant. The result is a rather uniform distribution of ε in the vertical.

When shear and buoyancy production combine, the result is a non-linear mix of both effects. However, Moeng and Sullivan (1994) suggest that the effects may be linearly combined using the appropriate scaling. This combined scaling will be used later in the paper.

Equation (1) and Figs. 1 and 2 describe the complete mean TKE budget of all eddy sizes. Aircraft do not respond to all the turbulence but to only a portion of the turbulence spectrum (Vinnichenko et al. 1980). In large eddies aircraft move smoothly up and down with the flow. On the other end of the spectrum, aircraft hardly feel the small eddies because they create such light loads. Aerodynamic parameters, such as aircraft design, altitude, speed, and weight also determine how much buffeting an individual aircraft receives.

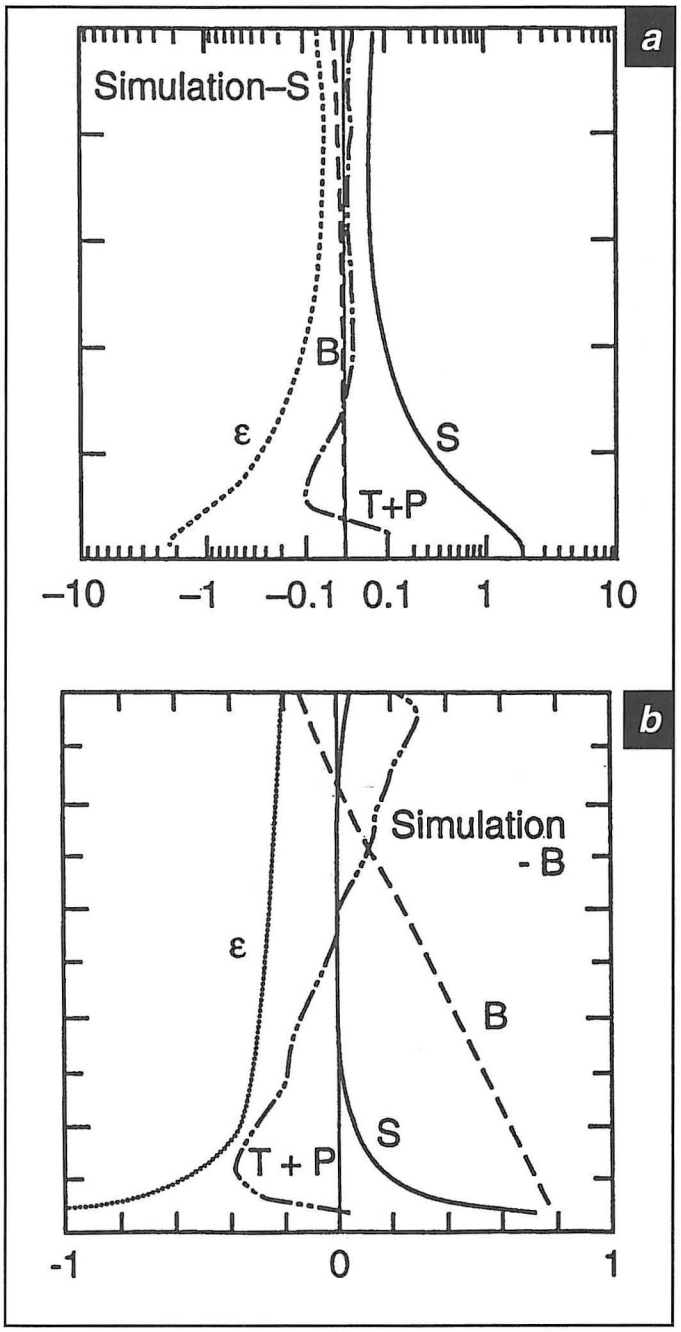


Fig. 2. Vertical distributions of the terms in the mean TKE budget obtained from Moeng and Sullivan's (1994) large eddy simulations: a) shear-dominated simulation and b) buoyancy-dominated simulation. S is the shear production, B is the buoyancy production, (T+P) is the sum of the turbulent and pressure transport terms, and ε is the turbulent dissipation.

3. A Simple TKE Equation

Applying equation (1) in the planetary boundary layer for aviation meteorology involves some simplifications. First, it cannot be solved analytically but, in practice, is solved statistically by finding relationships for the terms in the angle braces. There are equations for these second-order terms, but they involve unknown terms of the third-order. These third-order terms have equations that contain fourth-order terms, and so forth; it is impossible to close (1). The higher order terms must be parameterized in terms of known quantities.

The simplest scheme is a first-order closure method called the flux gradient method or *K*-theory. Although higher order closure schemes have been shown to be necessary to describe the structure of the turbulent boundary layer, many times first-order closure is sufficient (Garratt 1992). In aviation meteorology the time and space scales are large, and a complete description of the turbulent boundary layer is not crucial. It is assumed that turbulence is analogous to diffusion, i.e., that the average of turbulent fluctuations of a quantity, a term in angle braces, is proportional to the mean gradient of that term. Thus

$$-\langle v'w' \rangle = K_m \frac{\partial \overline{v}}{\partial z} \tag{2}$$

and

$$-\langle w'\Theta' \rangle = K_h \frac{\partial \overline{\Theta}}{\partial z} \tag{3}$$

in which K_m and K_h are positive eddy diffusivities for momentum and heat, respectively. For time periods and spatial resolutions much larger than individual eddies, $\partial e/\partial t = 0$, and the dissipation, ε, and the production are in equilibrium, i.e., what is produced eventually is dissipated. Substituting (2) and (3) into (1) and ignoring transport terms

$$\varepsilon = K_m \left(\frac{\partial \overline{v}}{\partial z} \right)^2 - K_h \frac{g}{\overline{\Theta}} \frac{\partial \overline{\Theta}}{\partial z} \tag{4}$$

The transport terms can be important, especially in a convective boundary layer, but K_h can be chosen to account for them, as will be shown below. This equation describes the spatial and temporal mean TKE production/dissipation in a layer.

Except for the eddy diffusivity values, (4) is in terms of mean quantities that can be measured. Note that wind shear always causes positive TKE production, no matter whether the wind is increasing or decreasing with height. Note also that positive buoyancy production occurs only when the potential temperature decreases with height, i.e., with a superadiabatic lapse rate. In stable conditions this term is negative and suppresses TKE production. Clearly, there is a potential problem with (4). Turbulent dissipation is always positive, and the buoyancy production term can be negative enough to produce negative TKE production. However, since it is assumed that dissipation and production are in equilibrium, it follows that, in this simple equation, TKE production/dissipation cannot be negative and is actually zero whenever the sum of the production terms is less than zero.

Furthermore, the ratio of the two production terms defines the Richardson number, *Ri*:

$$Ri \equiv \frac{\frac{g}{\overline{\Theta}} \frac{\partial \overline{\Theta}}{\partial z}}{\left(\frac{\partial \overline{v}}{\partial z} \right)^2} \tag{5}$$

The Richardson number's numerator is the stability as measured by the Brunt-Väisälä frequency squared (BVSQ). The denominator is the wind shear squared (WSHRSQ). Substituting into (4)

$$\varepsilon = K_m\left(\frac{\partial \bar{v}}{\partial z}\right)^2 - K_h(Ri)\left(\frac{\partial \bar{v}}{\partial z}\right)^2 \tag{6}$$

or

$$\varepsilon = K_h\left(\frac{\partial \bar{v}}{\partial z}\right)^2\left(\frac{K_m}{K_h} - Ri\right) \tag{7}$$

The ratio of the eddy viscosity (K_m) to the eddy thermal diffusivity (K_h) is called a turbulent Prandtl number (Pr). Inspecting (7), only when $Ri < Pr$ will the TKE production be positive. Therefore, one's choice of Pr determines the range of conditions for which the TKE equation will diagnose turbulence production. Mellor and Yamada (1982) suggest $Pr \approx 0.8$ for numerical modeling of boundary layers. Others, summarized in Garratt (1992), suggest Pr is quite variable in the boundary layer.

In real fluids, the local Richardson number must be less than 0.25 for turbulence to begin. This can be shown theoretically (Miles and Howard 1964) and experimentally (Thorpe 1969). Cursory inspection of (7) suggests that $Pr = 0.25$ if positive TKE production is related to positive turbulence. However, turbulence may occur in a "thick" layer in which $Ri > 0.25$. Thinner layers within the thick layer may locally have Ri below the critical value of 0.25 due to unmeasurable forces. In the mean there is positive turbulence with $Ri > 0.25$, but it is intermittent (Kondo et al. 1978). The greater the Ri in the thick layer, the less frequent the turbulence events. The mean TKE production is the integration of all the turbulent events in space and time within the thick layer. Therefore, turbulence is a probabilistic function of Ri, and the Pr defines the upper limit of Ri in which the probability is greater than zero. As the thick layer Ri approaches 0.25 from higher values, the probability of turbulence at any one time and at any one point approaches 100%.

The remainder of this paper describes an experiment comparing a constant $Pr = 0.25$ with a constant $Pr = 0.8$ in a TKE production equation for aircraft turbulence. The questions are twofold: 1) Given that aircraft do not respond to the entire turbulence spectrum, does TKE dissipation, equation (4), relate to aircraft turbulence as observed in the boundary layer? 2) If so, which Prandtl number gives the best results?

4. Boundary Layer TKE Production

The first task in implementing the simple TKE equation is to estimate values for the eddy diffusivities, K_m and K_h. Three, not very rigorous approaches are usually taken: (1) prescribing K-values, (2) prescribing K-profile shapes, and (3) prescribing K-dynamics (Garratt 1992). The first is the easiest and simplest. For example, the familiar Ekman spiral of the boundary wind is derived when K is set constant (Garratt 1992). The second is popular and assumes that the K-profile is such that there is a TKE balance between the local dissipation and the local production. In other words, a good K-profile will replicate the TKE production profiles in Fig. 2. In the third case, K is related to the TKE itself, and the closure is called "one-and-a-half."

The K-values estimated for operational aviation weather forecasting require additional consideration. Vinnichenko et al. (1980) note that turbulence felt by an individual aircraft is regarded as a set of discrete air gusts that are independent of one another. Statistical averages of turbulent profiles are fine for describing the mean turbulence in a layer, but an aircraft may feel something quite different from the average. Figures 1a and 1b are snapshots of instantaneous turbulence production, while Figs. 2a and 2b are the mean of all the snapshots. The AWC's task is to advise aircraft of potentially hazardous turbulence. Therefore, a turbulence product for the AWC must assume that at any moment, the worst turbulence in a boundary layer could be anywhere in that boundary layer.

One can compute either K-value and infer the other by the chosen constant Prandtl number. Garratt (1992) gives a simple formula for the maximum K_m in a boundary layer

$$K_m = kv_*h \tag{8}$$

where v_* is an appropriate scaling speed, and h is the height of the boundary layer. The constant, k, varies with stability and is about 0.06 for neutral conditions and 0.03 for moderately stable conditions. In the convective boundary layer $k \approx 0.05$. The scaling speed is about 0.5 m s^{-1} for pure shear conditions, 2.0 m s^{-1} for pure buoyancy conditions, and about 1.2 m s^{-1} for mixed conditions (Moeng and Sullivan 1994). For completeness, $K_h = K_m / Pr$.

The actual values of K_m and K_h used to compute TKE production are not that important. What is important for aviation turbulence diagnosis is to correlate the value of the TKE production in (4) to the turbulence intensity felt by the aircraft. For each workday between April 1996 and April 1997, "random" pilot reports of turbulence over the contiguous United States below 3000 feet (about 900 m) above ground level (AGL) were gathered at the AWC. The method was to gather one randomly located report, if available, from each of four categories, SEVERE, MODERATE, LIGHT, and SMOOTH. Pilot perception of aircraft turbulence is very subjective, and, as pointed out earlier, some of the additional objective factors influencing turbulence severity are aircraft speed and type. Because of the many variables, all aircraft intensities were treated equally as reported. Reports were gathered plus-or-minus one hour at 1500 UTC, 1800 UTC, 2100 UTC and 0000 UTC which are times when there are usually sufficient pilot reports for random sampling.

To compute TKE production from (4), the wind and temperature at the top of the layer were at a level 90 mb above the surface from the Rapid Update Cycle (RUC) operational numerical model, and surface values at the bottom of the layer were from the hourly Rapid Update Cycle surface (RUCS) objective analysis. The eddy viscosity, K_m, was constant and was computed from (8) assuming $k = 0.05$, $v_* = 1.2$ m s^{-1}, and $h = 900$ m. The eddy thermal diffusivity, K_h, was inferred from two different

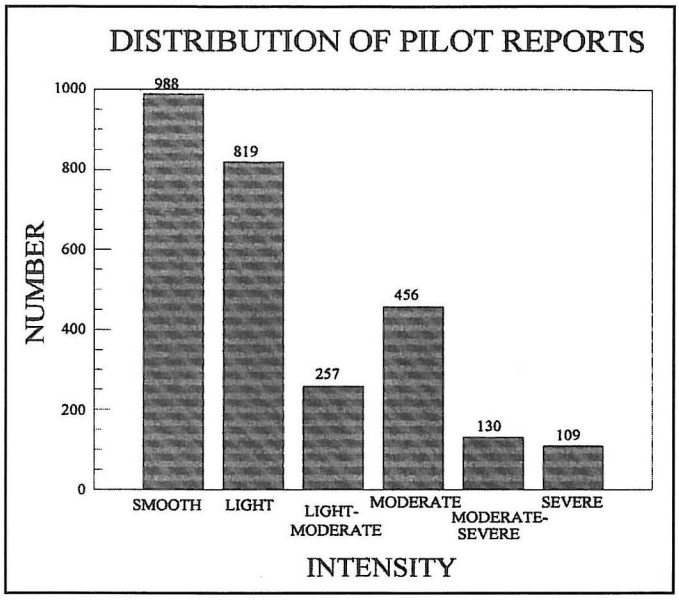


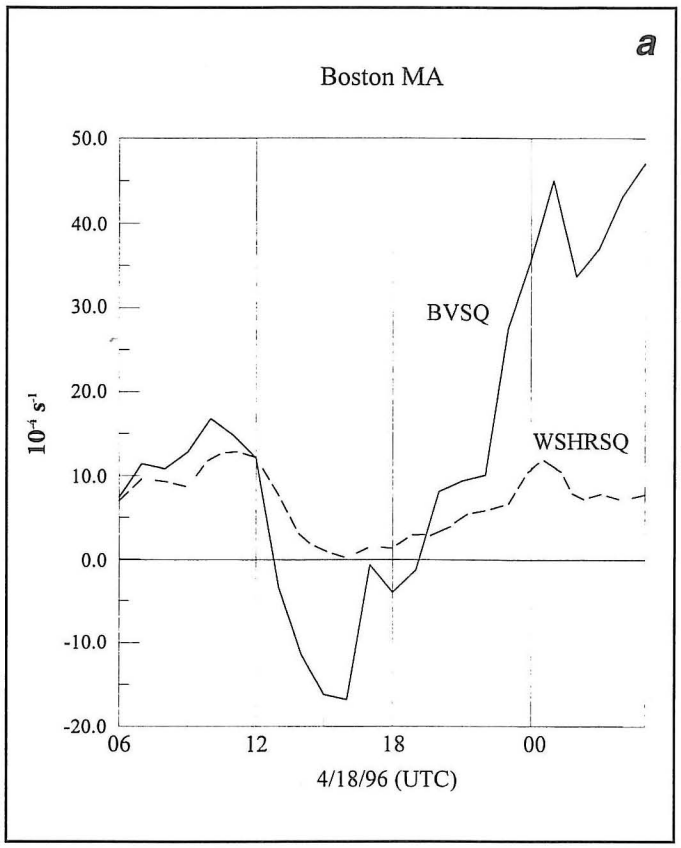
Fig. 3. The intensity distribution of the 2759 turbulence pilot reports in the experiment's database.

Prandtl number assumptions, Pr = 0.25 and Pr = 0.8. With K_h constant with height, (4) computes a constant buoyant TKE production throughout the depth, not the decrease with height seen in Fig. 2b. However, the sum of B and (T + P) is nearly constant with height. Therefore, a constant eddy thermal diffusivity accounts for the important transport terms ignored in deriving (4). A constant K_m was justified by the assumption above that the maximum shear TKE production can occur anywhere within the boundary layer, although it is more likely to occur in the lower portion of the boundary layer.

Each pilot report was matched with the maximum RUC grid point TKE production of the four grid points surrounding the aircraft location. There were 2759 comparisons of TKE production with aircraft turbulence intensity. The distribution of the turbulence pilot reports is shown in Fig. 3.

The two Prandtl number assumptions can give quite different TKE production values. One example is from 18 April 1996 in the vicinity of Boston MA. Figure 4a shows how the BVSQ and the WSHRSQ varied during the day. Note that the boundary layer became convective early in the day as the BVSQ < 0.0 and WSHRSQ → 0.0 between 1400 UTC and 1800 UTC. A front passed through the Boston area about 1800 UTC and increased the BVSQ substantially. In Fig. 4b, while both the Pr = 0.25 version and the Pr = 0.8 version show an increase of the TKE production after 1200 UTC and a decrease after 1800 UTC, the Pr = 0.25 version showed more dramatic changes. Peak TKE production values were 0.012 J s^{-1} for the Pr = 0.8 version versus 0.036 J s^{-1} for the Pr = 0.25 version. The different values were due to the computed buoyancy TKE production being 3.2 (0.80/0.25) times larger in the Pr = 0.25 version than the Pr = 0.8 version. The correlation between the TKE production and the aircraft turbulence intensity in both cases is good.

In the aggregate, the computed TKE production from both Prandtl number relationships showed correlations



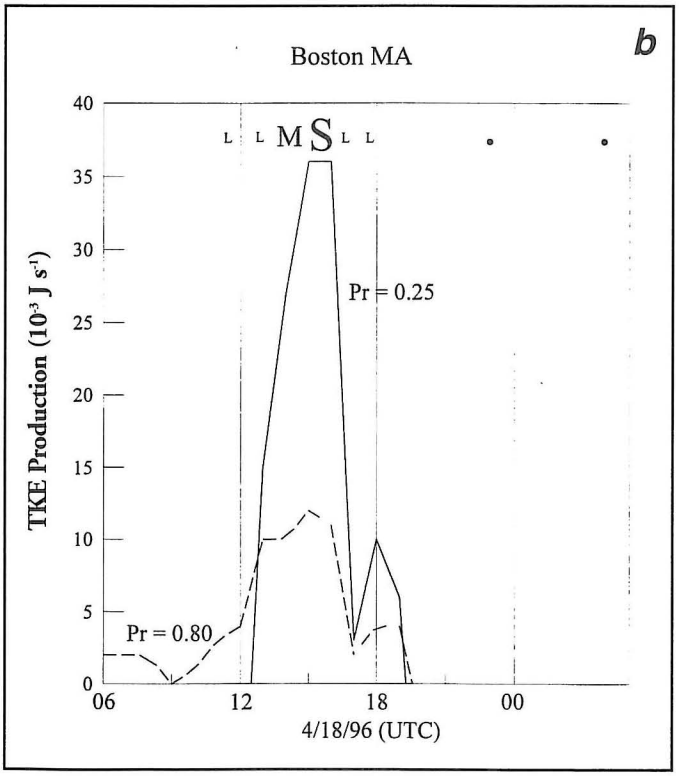


Fig. 4. Near Boston, Massachusetts, on 18 April 1996. Time sequence of a) BVSQ (solid) and WSHRSQ (dashed) and b) the TKE productions from Eq. (4) as outlined in the text using Pr = 0.25 (solid) and Pr = 0.80 (dashed). Pilot reports near Boston appear along the top (S for SEVERE, M for MODERATE, L for LIGHT, and a dot for SMOOTH).

Table 1. Binary joint distribution tables for TKE production and turbulence severity for the $Pr = 0.25$ TKE and the $Pr = 0.8$ TKE equations at the thresholds that maximize the Heidke Skill Score (HSS). Units of TKE production are J s^{-1}.

		$Pr = 0.25$		$Pr = 0.8$	
SMOOTH/LIGHT					
		Threshold = 0		Threshold = 0	
		FORECAST			
		yes	no	yes	no
OBSERVED	yes	885	523	716	451
	no	103	1248	272	1320
		HSS = .549		HSS = .452	
LIGHT/MODERATE					
		Threshold = 0.016		Threshold = 0.007	
		FORECAST			
		yes	no	yes	no
OBSERVED	yes	374	166	458	391
	no	321	1898	237	1673
		HSS = .494		HSS = .437	
MODERATE/SEVERE					
		Threshold = 0.035		Threshold = 0.021	
		FORECAST			
		yes	no	yes	no
OBSERVED	yes	38	92	55	218
	no	71	2558	54	2432
		HSS = .287		HSS = .245	

with aircraft turbulence intensity in the database. Skill scores computed from the joint binary distributions of pilot report observations and forecasts were maximized to find TKE production thresholds for turbulence intensity. Table 1 shows the distributions for the threshold that maximized the Heidke Skill Score (HSS) (Marzban 1998) for each intensity and each Prandtl number equation. The combinations of positive HSS and increasing threshold as the turbulence intensity increased indicate good skill in using TKE production equations for aircraft turbulence diagnostics and forecasts. The better HSS for the $Pr = 0.25$ equation for all intensity thresholds is surprising. Since the higher Pr diagnoses larger areas of positive TKE and since one of the assumptions was that aircraft may feel the worst turbulence in a layer, even though it is intermittent, one would expect the $Pr = 0.8$ to have better skill. Apparently the turbulence intensity is more accurately diagnosed when the probability is near 100% for turbulence. One reason for this may be that with a higher Pr, the computed TKE production may be the mean of a large number of weak events or a small number of stronger events. The skill scores apparently reflect the uncertainty inherent in the higher Pr assumption.

5. Conclusions

Turbulence forecasting techniques to date have, at best, only indirectly pointed to the atmospheric conditions in which turbulence actually develops. At worst, they have ignored the principles of turbulence generation for which even a simple TKE production equation accounts. The results of the AWC boundary layer turbulence study have shown that aircraft turbulence intensity in the boundary layer is related to simply-computed TKE production. Since the TKE equation is a more direct approach because it quantitatively determines the amount of turbulence, its value to forecasters is readily apparent. The results from the experiment in using a $Pr = 0.25$ in the boundary layer instead of more "traditional" higher values show that the TKE productions do not suffer because of the more restrictive conditions and are actually improved. The simple TKE equation is routinely computed on numerical forecast model output in the boundary layer at the AWC for forecaster guidance.

The TKE equation simplifies and focuses turbulence forecasting conceptually. There are only two sources for positive TKE production, wind shear and stability. A forecaster need only concentrate on how these two will change with time.

Acknowledgments

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Don McCann has been a research meteorologist in the Aviation Weather Center since 1991. Previously he was an aviation forecaster in the AWC's predecessor, the National Aviation Weather Advisory Unit. His research encompasses many aviation hazards, including icing, turbulence, and thunderstorms. He has a B.S. degree (1971) and a M.S. degree (1975) from the University of Missouri.

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