

AIRBORNE VERIFICATION OF ATMOSPHERIC TURBULENCE USING THE RICHARDSON NUMBER

Christopher C. Widseth¹
and
Dean A. Morss

Creighton University
Environmental and Atmospheric Sciences Department
Omaha, Nebraska

Abstract

This project focused on limited validation of the Richardson number (Ri) as a tool to forecast clear air turbulence (CAT) at the flight levels of most commercial and military aircraft. The Richardson number in this study was derived based upon ten flights over the United States in 1995. Airborne turbulence observations were taken by the principal author and received from other aircraft in the flight profile area. The results, while not representing an all encompassing case study, none the less illustrate the potential for Ri as a useful tool in operational forecasting of CAT. Values of Ri near four, rather than the theorized values less than one, correlated to higher possibilities for turbulence. The critical value of Ri for forecasting CAT could be further refined through more data from additional flights. There is also a need for more accurate aircraft instrumentation and a denser radiosonde and wind profiler network.

1. Introduction

A 1986 article in *Aviation Week and Space Technology* (AWST) reported that turbulence is the largest single cause of weather-related commercial air carrier accidents, (vol.103, pg.130-131). Further evidence of the significance of airborne turbulence is seen in a recent National Transportation Safety Board article (NTSB 1998) where it was reported that the only U. S. airborne commercial aviation fatality was the result of an encounter with turbulence. This flight phenomenon is so widespread that it occurs on one in every four flights from Atlanta to Los Angeles (AWST 1994). Military mission failures, personnel injury, and aircraft damage (Bender et al. 1976) have been attributable to turbulence. Between 1962 and 1974, airborne turbulence was a contributing factor in 189 out of 450 weather-related incidents. Between 1982 and 1984, six transports flying at cruise altitude (typically between 9,000 and 12,000 m) encountered severe turbulence, resulting in 14 serious and 64 minor injuries (AWST 1986).

This project examined clear air turbulence (CAT) events, defined here to be those turbulent events occurring above 5,000 meters, clear of clouds and removed from any mechanically generated terrain turbulence (AWST 1989). Conventional definitions of turbulence are found in many sources, each introducing its own unique characteristics. We find reference to the occurrence of turbulence above and below strong jet streams where huge eddies form and develop in clear air (Ahrens 1988; Stull 1988). Patchy sheets, stretching a few kilometers horizontally and from 10 to 300 meters vertically are reported by Balsley and Peterson (1981), Barat and Bertin (1984), and Keller (1981). These patches were then found to appear to drift along with the mean wind flow (Qin and Robinson 1992). This form of turbulence is widely believed to occur on strongly sloping, isentropic surfaces in strong baroclinic zones. Aircraft detection of these narrow zones defined by a thickness of a few hundred meters and width of a few kilometers (Woods 1969; Bender et al. 1976) is not an easy task. Generally, aircraft are allowed to fly on pressure surfaces separated by a minimum of 1,000 feet or 300 meters, thus aircraft flying closer than 100 meters is illegal and highly dangerous. Consequently, a single aircraft cannot measure wind speeds on adjacent pressure levels unless that aircraft is in a scheduled climb or descent. CAT occurrence even has annually preferred geographic regions (AWST 1994) where high-risk zones are found near the merger of the subtropical and polar jet which migrate north and south as the seasons change.

Previous studies of turbulence have focused on the phenomenon in the lower levels of the atmosphere, i.e., at altitudes below 700 meters (Sethuraman and Raynor 1980; Ian and Robinson 1997). These studies provide limited initial applicability to flight forecasting situations as modern commercial airliners and military aircraft spend by far the majority of their flying hours above this lower boundary layer. This project thus focused on the occurrence of turbulence in those levels of the atmosphere where most aviation activity occurs. Previous studies have relied on rawinsonde data nearest in time to the turbulence event; this can be as much as six hours before or after observation (Waco 1970). Kennedy and Shapiro (1980) used wind speeds, wind direction, and potential temperatures from rawinsonde data, Boucher (1973) also used rawinsonde data. Unfortunately, rawinsonde data does not provide the time and space resolution needed to

¹Current affiliation United States Air Force, Headquarters, Air Mobility Command, Scott Air Force Base, Illinois. "The views expressed in this article are those of the authors and do not necessarily reflect the official policy or position of the United States Air Force, Department of Defense, or the US Government."

Table 1. Turbulence Forecasts Given Wind Speed and Shear

Wind Speed at Flight Altitude (knots)	Amount of Speed Shear per 1000 Feet (knots)	Turbulence Forecast to be Issued
40-60	5-7	None
	8-10	Light
	11-20	Light to Moderate
	21-30	Moderate
	31-50	Moderate to Severe
	50+	Severe
61-120	5-7	Light
	8-10	Light to Moderate
	11-20	Moderate
	21-30	Moderate to Severe
	31-50	Severe
	50+	Severe to Extreme
120+	5-7	Light
	8-10	Light to Moderate
	11-20	Moderate
	21-30	Moderate to Severe
	31-50	Severe
	50+	Extreme

from US Air Force, *Air Weather Service Regulation 105-27*.

accurately detect, let alone forecast, small patches of turbulence. The US National Weather Service (NWS) currently operates a total of 102 rawinsonde stations nationwide, with observations taken twice daily. It is quite obvious that this spatial and temporal distribution is less than ideal for detecting and forecasting a phenomenon with the small scales associated with clear air turbulence. A CAT index (based upon data averaged monthly, seasonally and annually) will hopefully provide some assistance for regions where severe turbulence is strongest (AWST 1994). The basis for this CAT index is a combination of the intensity of vertical wind shears between two altitudes and the amount of deformation and compression that happens between the two levels.

This project used real-time wind data from wind profilers and airborne aircraft to infer where turbulence was occurring. Earlier studies merely used aircraft as detectors of turbulence, not as sources of real-time wind information. This real-time data, with its increased temporal and spatial resolution, should support efforts to determine if the dynamic and thermodynamic processes considered in the gradient Ri are valid criteria for forecasting turbulence (Ray 1986).

2. Turbulence Forecasting Approaches

Aircraft turbulence is defined subjectively in guidelines set forth in the Department of Defense Flight Information Handbook (DMA Aerospace Center 1995). Light turbulence or light "chop" is defined in the handbook as "turbulence that momentarily causes slight, erratic changes in altitude." Under these conditions, occupants may feel a slight strain against seat belts, food service can be conducted with little difficulty, and unsecured objects would only be displaced slightly. Moderate turbulence causes changes in aircraft altitude or attitude, or both, pilots still have control of the aircraft. Occupants

would feel definite strains against seat belts, while walking and food service become difficult.

At present, Air Force weather forecasters issue turbulence warnings emphasizing areas where significant wind shear exists or is forecast to occur. Air Force Manual 51-12 Vol. II delineates other potential turbulent areas including troughs aloft, upper-level closed cyclonic circulations, large temperature gradients, and vertical wind shears. A nomogram from Air Weather Service Regulation 105-27 enables the development of warnings for turbulence generated by wind speed shear. This technique initially includes the wind speed at altitude and the amount of vertical wind shear per thousand feet. Given this input information, turbulence forecasts are developed based upon the parameters shown in Table 1 from Air Weather Service Regulation 105-27.

Forecasts of moderate turbulence are typically issued based upon the presence of areas of cold or warm air advection, positive vorticity advection, cyclogenesis, and the degree of curvature exhibited by constant height contours. In addition, moderate turbulence is normally forecast to occur within 700 meters above and 2,000 meters below the tropopause, particularly near the jet stream. Further, changes in the horizontal wind direction may itself be sufficient to forecast moderate turbulence near ridges and troughs. The turbulence generated by the change in horizontal wind direction at a constant height is not accounted for in Ri ; only changes in the wind speed and direction measured between different heights are included in the calculation (Lee et al. 1979). Wind shear generated turbulence has been validated via airborne aircraft observations. Wind direction changes of 10-15 degrees, or velocity changes of 10-20 knots, or both, that occur in less than one minute (as observed on the Inertial Navigation System (INS) wind display) usually result in turbulence.

3. The Richardson Number and Turbulence

The Richardson number (Ri) describes the atmosphere's ability to sustain turbulence produced through other means by quantifying the ratio of static stability to vertical wind shear. It is defined as

$$Ri = \frac{(g/\bar{\theta})(\partial\theta_m/\partial z)}{((\partial\bar{u}/\partial z)^2 + (\partial\bar{v}/\partial z)^2)} \quad (1)$$

where g is gravity, θ_m is potential temperature, u is the x-component of velocity, v is the y-component of velocity, z is the vertical coordinate (Holton 1992), and $\bar{\theta}$ denotes the average potential temperature in a vertical layer. This gradient Ri is commonly defined as the ratio of buoyancy to vertical wind shear (Stull 1988).

The value of Ri does not indicate turbulence intensity; it merely indicates if turbulence can be sustained. It therefore represents a necessary but not sufficient condition for turbulence occurrence (Oard 1974). The turbulence we are seeking to forecast is generated in one of two ways. First, surface heating can create an unstable temperature lapse rate near the ground and convective overturning consequently occurs. We also must account for

turbulence generated by dynamic instability from wind shear, represented by an energy conversion between the mean flow and turbulent fluctuations. The Richardson number provides a means to estimate turbulence based upon the relationship between areas of significant CAT and areas of statically stable air with strong vertical shears (Keller 1981). Finally, Ri does not appear to be affected by global atmospheric phenomena, such as an El Niño, since only 0.3% of the variability is associated with yearly variations. Location (latitude and longitude), altitude and season can adequately explain 24-50% of the variability of Ri (Murphy et al. 1982).

However, in a statically stable layer, turbulence can only persist if the mechanical production can overcome the damping effects of stability and viscous dissipation. When Ri is unity, the buoyancy term is removing energy as fast as it is introduced by wind shear; the turbulence cannot maintain itself. In such a case, the minimal Ri does not give the criteria for the onset or disappearance of turbulence; an instability or another outside source must first generate the disturbance (Lumley and Panofsky 1964). Observational and laboratory research (Stull 1988) both conclude that mechanical production is intense enough to sustain stable layer turbulence (with an Ri less than 0.25).

It thus seems reasonable to propose that a critical Ri of 0.25 might be used as a lower threshold provided the meteorologist had data on the actual changes in potential temperature and wind at a point, i.e., via a laboratory situation. In the free atmosphere of aviation operations, wind and potential temperature gradients cannot be directly measured, but must be estimated from widely dispersed observational parameters. Consequently, a larger gradient Ri has been accepted as the threshold for determining turbulence; most commonly, the practice is to use a value near unity as the precise indicator of moderate turbulence. Murphy et al. (1982) found that the lowest values of Ri are correlated with the highest probability of CAT and that the turbulence is usually centered near nine kilometers above mean sea level, particularly near the jet stream. Many flights in this study were near this height.

4. Data Collection

Aircraft turbulence measurements were obtained from an unmodified Boeing EC-135C, four-engine jet aircraft. Wind speed and direction, as well as navigation positions were recorded using the aircraft's Inertial Navigation System (INS). Using a gyro-stabilized platform and gimbal assembly that is electronically controlled, the INS provides horizontal and azimuth reference regardless of the aircraft attitude. The manufacturer of the INS has specified a positional accuracy of 1.28 ms^{-1} for this navigational system (Boeing Technical Order 1C-135(E)C-1 1995), although accuracy's of 0.10 ms^{-1} or less have been documented (Lemone and Pennell 1980). The Boeing EC-135C is also able to measure outside air temperature (not used as a data input in this study) through a pitot static tube. Bender et al. (1976) have previously determined, through use of a Boeing 747, that temperature gradients obtained from aircraft temperature sensors are not well

correlated with CAT. In support of this analysis, wind and potential temperature data were obtained from radiosonde and wind profiler data via the Unidata McIDAS (Man Computer Interactive Data Access System) weather computer system at Creighton University.

Turbulence reports were gathered from the primary aircraft used for the project as well as from other nearby commercial aircraft that reported turbulence. Aircraft submitted reports of turbulence encounters to the air traffic control agency as the aircraft traversed each sector. Thus, although the exact locations of these other aircraft were not known, it can be safely assumed that they were within 100 kilometers of the primary aircraft since air traffic control sectors are roughly 100 kilometers across.

5. Methodology

The use of only the wind profiler data or only the rawinsonde data does not adequately address the problem. Large errors in the horizontal winds, as much as 20 knots, may arise when trying to measure wavelengths of 5 to 15 km. However, these large errors become negligible when averaging periods of wavelengths that are 10 km for typical profiler geometry. Keller (1981) determined that rawinsonde data alone should not be used to detect the existence of CAT because the time differences between launches may well lead to different trajectories and vertical differences that are too coarse to fully support CAT detection or forecasting. For this project, rawinsonde data were only used to derive potential temperatures.

The Richardson number was calculated using a variety of data sources. In-flight data were tabulated when the aircraft was near a wind profiler station regardless of the observed turbulence conditions. Winds and altitudes were interpolated from displayed wind profiler data after the flight. When turbulence was encountered, the nearest and most current RAOB data were obtained and used in conjunction with winds from the profiler station; this only occurred if the aircraft did not transition between two different altitudes. In all cases, the potential temperatures used were taken from the most recent and nearest RAOB since in-flight temperatures are subject to multiple interpolations and are also ineffective in providing useful information towards the determination of Ri .

Wind readouts were recorded on the aircraft every 300 meters in the vertical; conversely, RAOB and wind profiler data can easily have vertical resolutions greater than 1,000 meters. Mahrt (1985) stated that when the vertical resolution is 20-70 meters, the gradient Ri would have to be 1.3 to indicate turbulence and approximately 9 to indicate no turbulence. For vertical sampling resolutions greater than 70 meters, the numbers distinguishing turbulent conditions would increase since larger vertical distances increase Ri . The critical Ri for distinguishing between turbulent and nonturbulent conditions will probably increase as a result of this type of data collection.

A detailed example of Ri calculation using aircraft and RAOB data for 7 February 1995 is illustrated in Table 2, Richardson Number Worksheet. As the aircraft flew over

Table 2. Richardson Number Worksheet

Date: 7-Feb-95 Time: 1710 UTC Location: Fairbury NE				
Aircraft Height (m)	8380	Upper Level Wind Speed (kt)	137	
Upper Level Potential Temp (θ)	321	Upper Level Wind Speed (ms^{-1})	70.5095	U-Component of Lower Wind 33.4536
Upper Level Potential Temp Height (m)	8780.4	Upper Level Wind Height	8380	V-Component of Lower Wind -57.9427
Lower Level Potential Temp (θ)	304.5	Upper Level Wind Direction (deg)	330	U-Component Wind Shear Term Squared 0.00014
Lower Level Potential Temp Height (m)	7621	Lower Level Wind Direction (rad)	5.75956	V-Component Wind Shear Term Squared 0.00043
Gravity divided by Average Potential Temp	0.03136	Lower Level Wind Speed (kt)	130	Total Wind Shear (denominator) 0.00058
Potential Temp Lapse rate (θ_m)	0.01423	Lower Level Wind Speed (ms^{-1})	66.9068	Turbulence Reported Smooth
Wind Source (A-Aircraft; P-Profiler)	A	Lower Level Wind Height	8230	Source of Turbulence Report A
Upper Level Wind Direction (deg)	330	U-Component of Upper Wind	35.255	Richardson Number 0.77359
Upper Level Wind Direction (rad)	5.75956	V-Component of Upper Wind	-61.0627	

the Fairbury, Nebraska, wind profiler site at 1710 UTC, no turbulence was experienced, thus "smooth" conditions were recorded in the third column. The altitude of the aircraft at the time of the report was 27,500 feet MSL (8,380 meters). The aircraft did change altitude while over Fairbury, dropping down to 8,230 meters. Since the altitude of the aircraft did change, a vertical shear of the horizontal wind could be locally determined from the INS wind readout on the aircraft.

The average potential temperature and the vertical gradient of potential temperature were determined post flight from the most recent and closest RAOB sounding. In this case, the sounding used was taken at 1200 UTC 7 February 1995 from Valley, Nebraska. Potential temperature values from the RAOB, encompassing the flight profile, were determined to be 321°K (8,730 m) and 304.5°K (7,621 m), thus giving us the terms needed to calculate the numerator of Ri.

The u and v components of the winds at or near flight levels form the basis of the wind shear components of the equation. In this case, the wind at 8,380 meters was 330°/137 knots and the wind at 8,230 meters was 330°/130 knots. These values were converted to speed in meters per second, and zonal and meridional wind components were computed. We thus have the final terms

needed to arrive at the gradient Ri for this flight segment.

6. Results

Ten flights helped provide data for this project; 49 observations were collected: 22 for smooth air, 19 for light chop and 8 for moderate turbulence. As shown in Table 3, the values of Ri were widespread for all turbulent conditions. Values of Ri for "smooth" flights varied from 0.135 to 104.5; light chop values ranged from 0.10 to 60.9; and moderate turbulence conditions ranged from 0.047 to 16.37. Average Ri values, accounting for all data sources, for smooth conditions, light chop, and moderate turbulence were 16.137, 6.377 and 3.911, respectively. Interestingly, lower average (and the smallest range in calculated values) Ri values occurred for moderate turbulence conditions. These average values are much higher than the theorized value of one. It would seem reasonable, based upon these flights, to forecast Ri at the range of four to six, or less, to predict turbulence during flight operations.

The data shown in Table 3 suggest that current aircraft and real-time wind profiler data are valid inputs for Ri development and the ultimate prediction of turbulence.

One shortcoming of this technique lies in the difficulty of getting a true measurement of the potential temperature gradient, especially since the current temperature instruments on the aircraft do not accurately measure temperature (Bender et al. 1976). Thus the only source used for potential temperature gradient determination was the data from the RAOB nearest the flight path, often several kilometers from the nearest profiler station. A further difficulty arises in that rawinsondes are not released as frequently as they were in previous years.

We find Ri values for moderate turbulence are smaller when derived from aircraft source measurements than from either wind profiler, RAOB, or combination data sets from the aircraft/profiler. When only aircraft data were used for moderate turbulence calculations, the average Ri was 0.348, compared to a wind profiler-based average value of 3.128. However, when considering aircraft data against profiler data for light turbulence, the average values are nearly the same (7.049 and 6.077 respectively).

There was a large difference when comparing profiler data against aircraft source data for measuring no turbulence conditions. In non-turbulent conditions, the average Ri for profiler data was 36.56112 versus 3.566 for aircraft data. A possible reason for this great disparity is that three of the large Ri based on profiler values (87.301

Table 3. Richardson Number Summary

OVERALL RICHARDSON NUMBER SUMMARY				
Turbulence Reported Data Source	None	Light	Moderate	Average
RAOB		0.49	1.74	1.11
Aircraft	3.57	7.05	0.35	3.65
Wind Profiler	36.56	6.08	3.13	15.26
Aircraft/Profiler	14.00	9.65	11.13	11.59
Overall Average (except RAOB data)	18.04	7.59	4.87	10.17

at 1949 UTC 13 February 1995; 104.489 at 2050 UTC 13 February 1995; and 20.978 at 2141 UTC 31 March 1995) occurred at altitudes in excess of 9,000 meters. This altitude would be in the stratosphere in the late winter. Potential temperatures increase rapidly in the stratosphere and thus higher values of Ri would result.

In summary, Ri has proven to be a useful predictor of flight turbulence. The data from this study indicates that a more realistic threshold for the turbulent Ri (using all sources of data) is closer to four than it is to the theorized value of 0.25. Further investigation, with more flights and associated data collection, will be necessary to validate these results. Of particular interest would be a study of the effect of scale in defining turbulent flow in a laboratory situation versus the turbulent flow noticeable via large aircraft.

7. Summary and Conclusions

This project was conducted to determine how Ri substantiates evidence of turbulence during flight operations. A secondary purpose was to determine if there was a range of Ri that validated operational use of the theoretical values. This project was unique in that no recent study had directly measured winds using an airborne platform at the altitudes of this study. Balsley, 1981 and Brooks, 1997 both incorporated aircraft-based data, but their altitudes were restricted to a boundary layer situation, i.e., below 1 kilometer. Further, wind profiler data had not been previously used to determine a relationship between Ri and moderate flight turbulence.

The data presents evidence that values of Ri associated with significant turbulence are lower when derived from wind profiler and aircraft sources. When the aircraft data is used, Ri for moderate turbulence are even lower than those from the profiler data because the aircraft is able to record wind changes over smaller vertical layers than profiler data. **The aircraft is able to more precisely measure the atmosphere and detect small patches of moderate turbulence, generally only ten to hundreds of meters in depth by kilometers wide.** The spacing of profiler instruments is presently on the order of hundreds of kilometers apart and the data itself is only resolved to every 1000 meters vertically. Thus profilers, as presently deployed, simply do not have the resolution to detect these moderate turbulence patches in support of military or commercial flight operations.

This project is presented to lay the groundwork for

future research that will more narrowly define the range of values of Ri associated with flight turbulence. It was unfortunate, only from a research point of view, that not one of the ten flights encountered severe turbulence. In addition, future studies should include vertically stacked aircraft flying through areas of moderate turbulence. Air refueling missions often involve two or more aircraft flying 150 meters apart vertically and one to two miles horizontally. Simultaneously measuring wind speed and direction from different aircraft flying through varied conditions would be a major step towards determining a true gradient Ri for turbulent conditions.

In-flight temperature gauges should not be discounted but held for future use when, ideally, airborne temperature measurements could be directly compared to the rawinsonde data during experimental flights. Additional profiler data would help studies like this tremendously. For this project, wind profiler data was often not available via the McIDAS system on flight days. Further resolution of the critical range of Ri would be possible if real-time wind profiler data were available at vertical intervals less than 1000 meters.

Another problem in the turbulence arena involves mathematics. An early goal for the statistical theory of turbulence was to find a finite, closed set of equations for quantities such as the mean velocity and the energy spectrum. This goal is now thought to be unrealistic. The current goal is to reduce the multiple degrees of freedom used to solve the analytical equations defining turbulent flow (Frisch and Orszag 1990).

Future operational applications of this study might use the reported winds and temperatures from the current rawinsonde network to construct Ri arrays. These might lead to objective turbulence forecasts for areas meeting the criteria for turbulence according to the Richardson number thresholds. This information could then be disseminated, perhaps via a public access Internet site such as that presently available from the Airline Dispatchers Federation at:

<http://www.dispatcher.org/brief/adfbrief.html>.

Authors

Christopher C. Widseth is an active duty officer in the United States Air Force stationed at Air Mobility Command Headquarters at Scott Air Force Base (AFB), Illinois. He is a flight navigator with over 2,500 hours in the EC-135 Looking Glass and KC-135 Stratotanker aircraft. He has Masters Degrees from both Creighton University (Atmospheric Science) and Embry-Riddle Aeronautical University (Aeronautical Science); he is also a licensed FAA Aircraft Flight Dispatcher.

Dr. Dean A. Morss has over 15 years of experience teaching at the graduate and undergraduate levels; presently as an Assistant Professor with the Environmental and Atmospheric Sciences Department at Creighton University. In addition to his teaching, Dr. Morss has over 30 years of experience in systems application and analysis in private industry and as a staff weather officer with the U.S. Air Force. He has earned two Masters Degrees (Aeronomy and Atmospheric Sciences) and his Doctorate (Atmospheric Sciences) from

the University of Michigan. His undergraduate degree (Bachelor of Electrical Engineering) is from the University of Minnesota.

References

- Ahrens, C.D., 1988: *Meteorology Today*. West Publishing Company, 581 pp.
- American Meteorological Society, *Glossary of Meteorology*, 1995., 638 pp.
- Aviation Week and Space Technology (AWST), Airborne Infrared System Provides Advanced Warning of Turbulence, 1989, 130, 130-131
- Aviation Week and Space Technology (AWST), National Oceanic and Atmospheric Administration Develops CAT Index, 1994, 140, 29.
- Aviation Week and Space Technology (AWST), Rough Air Slams China Eastern MD-11, 1993, 138, 36.
- Aviation Week and Space Technology (AWST), Scientists Using Satellite Imagery to Locate CAT, 1986, 125, 222-223.
- Balsley, B., and V.L. Peterson, 1981: Doppler-Radar Measurements of Clear Air Atmospheric Turbulence at 1290 Mhz. *J. Appl. Meteor.*, 20, 266-274.
- Barat, J., and F. Bertin, 1984: Simultaneous Measurements of Temperature and Velocity Fluctuations Within CAT Layers: Analysis of the Estimate of Dissipating Rate by Remote Sensing Techniques. *J. Atmos. Sci.*, 41, 1613-1619.
- Bender, M.A., H.A. Panofsky and C.A. Peslen, 1976: Temperature Gradients and Clear Air Turbulence Probabilities. *J. Appl. Meteor.*, 15, 1193-1199.
- Boeing Technical Order 1C-135(E)C-1, Change 95, 1995.
- Boucher, R.J., 1973: Mesoscale History of a Small Patch of Clear Air Turbulence. *J. Appl. Meteor.*, 12, 814 - 821
- Brooks, I.M., and D.P. Rogers, 1997: Aircraft Observations of Boundary Layer Rolls off the Coast of California. *J. Appl. Meteor.*, 36, 1834-1849.
- Brown, E.N., C. A. Friehe and D.H. Lenschow, 1983: The Use of Pressure Fluctuations on the Nose of an Aircraft for Measuring Air Motion. *J. Clim. Appl. Meteor.*, 22, 171-180.
- Chimonas, G., and D. Fua, 1984: Dispersion of Small-Scale Instabilities. *J. Atmos. Sci.*, 41, 1085-1091.
- Defense Mapping Agency Aerospace Center, *Department of Defense Flight Information Handbook*, 1995., 182 pp.
- Department of the Air Force, *Air Force Manual 51-12 Vol I: Weather for Aircrews*, 1990., 146 pp.
- _____, *Air Force Manual 51-12 Vol II: Weather for Aircrews*, 1992, 66 pp.
- _____, Air Combat Command, *Air Weather Service Regulation 105-27*, 1992.
- Duke, J.W., and J.A. Rogash, 1992: Multiscale Review of the Development and Early Evolution of the 9 April 1991 Derecho. *Wea. Forecasting*, 7, 623-635.
- Frisch, U., and S.A. Orszag, 1990: Turbulence: Challenges for Theory and Experiment. *Physics Today*, 43, 24-32.
- Grant, A.L.M., 1992: The structure of Turbulence in the Near-neutral Atmospheric Boundary Layer. *J. Atmos. Sci.*, 49, 226-239.
- Holton, J.R., 1992: *An Introduction to Dynamic Meteorology*, Academic Press, Inc., 511 pp.
- Keller, J.L., 1981: Prediction and Monitoring of Clear-Air Turbulence: An Evaluation of the Applicability of the Rawinsonde System. *J. Appl. Meteor.*, 20, 686-692.
- Kennedy, P.J., and M.A. Shapiro, 1980: Further Encounters with CAT in Research Aircraft. *J. Atmos. Sci.*, 37, 986-993.
- Lee, D.R., R.B. Stull, and W.S. Irvine, 1979: *Clear Air Turbulence Forecasting Techniques*. AFGWC/TN-79/001. Air Force Weather Agency, Offutt AFB, NE 68113., 73 pp.
- Lemone, M.A. and W.T. Pennell, 1980: A Comparison of Turbulence Measurements from Aircraft. *J. Appl. Meteor.*, 19, 1420-1437.
- Lumley, J.L., and H.A. Panofsky, 1964: *The Structure of Atmospheric Turbulence*. Interscience Publishers, 239 pp.
- Mahrt, L., 1985: Vertical Structure and Turbulence in the Very Stable Boundary Layer. *J. Atmos. Sci.*, 42, 2333-2349.
- _____, 1989: Intermittency of Atmospheric Turbulence. *J. Atmos. Sci.*, 46, 79-95.
- _____, and N. Gamage, 1987: Observations of Turbulence in Stratified Flow. *J. Atmos. Sci.*, 44, 1106-1121.
- Murphy, E.A., R.B. Agnostino, and J.P. Noonan, 1982: Patterns in the Occurrences of Richardson Numbers Less Than Unity in the Lower Atmosphere. *J. Appl. Meteor.*, 21, 321-333.

National Transportation Safety Board (NTSB), 1998. <http://www.nts.gov/>

Nelkin, M., 1992: In What Sense is Turbulence an Unsolved Problem? *Science*, 255, 566-569.

Oard, M.J., 1974: Application of a Diagnostic Richardson Number Tendency to a Case Study of CAT. *J. Appl. Meteor.*, 13, 771-777.

Panetta, R. L., 1993: Zonal Jets in Wide Baroclinically Unstable Regions: Persistence and Scale Selection. *J. Atmos. Sci.*, 50, 2073-2105.

Panofsky, H. A., and J. A. Dutton, 1984: *Atmospheric Turbulence*. John Wiley & Sons, Inc., 397 pp.

Qin, J., and W.A. Robinson, 1992: Barotropic Dynamics of Interactions between Synoptic and Low-Frequency Eddies. *J. Atmos. Sci.*, 49, 71-79.

Ray, P.S., 1986: *Mesoscale Meteorology and Forecasting*. American Meteorological Society, 793 pp.

Sekioka, M., 1970: Application of Kelvin-Helmholtz Instability to CAT. *J. Appl. Meteor.*, 9, 896-899.

Sethuraman, S., and G.S. Raynor, 1980: Comparison of Mean Wind Speeds and Turbulence at a Coastal Site and an Offshore Location. *J. Appl. Meteor.*, 19, 15-21.

Stull, R.B., 1988: *An Introduction to Boundary Layer Meteorology*. Kluwer Academic Publishers, 666 pp.

Thomson, D.W., R.L. Coulter and Z. Warhaft, 1978: Simultaneous Measurements of Turbulence in the Lower Atmosphere Using Sodar and Aircraft. *J. Appl. Meteor.*, 17, 723-734.

Waco, D.E., 1970: A Statistical Analysis of Wind and Temperature Variables Associated with High Altitude CAT (HICAT). *J. Appl. Meteor.*, 9, 300-309.

Woods, J.D., 1969: On Richardson's Number as a Criterion for Laminar-Turbulent Transition in the Ocean and Atmosphere. *Radio Sci.*, 1289-1298.