ON CALCULATING VERTICAL MOTIONS IN ISENTROPIC COORDINATES

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Abstract

Output from a mesoscale numerical model is used to calculate all three terms of the ω equation in isentropic space. Previously, it was thought that, in the presence of diabatic heating, the local pressure tendency term and the diabatic heating term would generally offset one another; that being the case, the transport term was expected to give an accurate representation of the vertical motion field. Calculations of all three terms of the isentropic ω equation for four case studies demonstrate that the local pressure tendency and diabatic heating terms do not always offset, and that the transport term alone, while typically of the correct sign, may underestimate the total ω_θ by two-thirds.

This study employed model output from simulations of four meso-α to synoptic-scale precipitation systems: two associated with strong extratropical cyclogenesis, and two forced primarily by jet streak dynamics. A representative isentropic surface was chosen, upon which each term of the ω_θ equation was calculated.

1. Introduction

Vertical motion ($\omega_\theta = \frac{dp}{dt}$) in isentropic space is typically estimated operationally by computing pressure transport on an isentropic surface. This relatively simple practice has been facilitated by the introduction of software packages such as GEMPAK and PC-GRIDDS, and now the National Weather Service's Advanced Weather Interactive Processing System (AWIPS), which can handle isentropic coordinates with ease. It is not uncommon to read Area Forecast Discussions, especially from Weather Forecast Offices in the National Weather Service Central Region, which mention isentropic vertical motion as estimated from the pressure transport term. However, estimating ω_θ in this way neglects both the local movement of the isentropic surface with respect to pressure ($\frac{\partial p}{\partial t}$), as well as the effect of diabatic heating/cooling on the isentropic surface. Several authors (e.g., Saucier 1955; Uccellini 1976; Homan and Uccellini 1987; Moore 1993) have justified this practice by assuming that the local pressure tendency and diabatic heating/cooling terms offset one another, and more recent studies have employed this approach effectively (de Coning et al. 1998; Moore et al. 1998). It has been noted that Uccellini (1976), following Saucier (1955), was quite clear about this approach being most effective where wind speeds were strong and directed along the pressure gradient. Yet, to attempt an estimate of ω_θ only in those situations would severely curtail the usefulness of isentropic analysis on a day-to-day basis. There are, in fact, many synoptic situations where these latter terms do not cancel and thus neglecting them results in an incomplete rendering of the total vertical motion field.

The purpose of this paper is to investigate the full ω_θ equation using output from a mesoscale numerical model and by calculating all of the terms in the expression. Our goal is to determine the scope and magnitude of the impact brought about by the neglect of terms in the ω_θ equation in a wider variety of synoptic and meso-α scale settings. Statistical analyses are provided for four cases wherein substantial precipitation, and thus diabatic heating (primarily in the form of latent heating), occurred and was predicted by the model.

2. Method

a. The ω_θ equation

The expression for vertical motion in isentropic space is an expansion of the substantive derivative of pressure with respect to time:

$$\omega_\theta = \frac{dp}{dt} = \left(\frac{\partial p}{\partial t}\right)_\theta + \vec{V} \bullet \nabla_\theta p + \frac{\partial p}{\partial \theta}\frac{d\theta}{dt} \qquad (1)$$

The first term on the right hand side (RHS) is the local time tendency of pressure on an isentropic surface. The second term represents the pressure transport on the surface, while the third term accounts for changes in potential temperature due to diabatic effects (latent heating/evaporative cooling, radiational heating/cooling, etc.) A positive sign in any of the three terms indicates descending motion, while negative values indicate ascent.

In an adiabatic environment ($\frac{d\theta}{dt} = 0$) parcels are bound to the isentropic surface on which they reside. So, vertical motions can be achieved in one of two ways: 1) through isentropic *surface* motion upward (to lower pressure) or downward (to higher pressure) as described by the local pressure tendency ($\frac{\partial p}{\partial t}$) term, or 2) through *parcel* motions along an isentropic surface leading to ascent (for motion toward lower pressure) or descent (for motion toward higher pressure) as given by the transport term ($\vec{V} \bullet \nabla_\theta p$). Thus, for the local tendency term, positive (negative) values point to descent (ascent) of the entire isentropic surface. It is important to note that this term can also measure changes in the height of the isentropic surface due to system translation, as well as in response to diabatic heating/cooling.

In the case of the transport term, positive values denote downglide (equivalent to cold air advection in p-space) while negative values indicate isentropic upglide (equivalent to warm air advection in p-space). This term is often used alone to estimate ω_θ for two reasons: first, because of its ease of calculation; secondly, due to the easy visualization and conceptualization of upglide and downglide. For the diabatic term, the static stability component $\left(\frac{\partial p}{\partial \theta}\right)$ is assumed to be always negative, in keeping with the generally stable, hydrostatic nature of the real (and modeled) atmosphere. Although the static stability helps to modulate the strength of the diabatic term, it is the total time tendency of potential temperature $\left(\frac{d\theta}{dt}\right)$ that controls the sign of the vertical motion. So, for added heat $\left(\frac{d\theta}{dt} > 0\right)$, the total diabatic term is negative, indicating ascent.

Another method of estimating ω_θ is to calculate the transport term only, but with the velocity modified to account for the storm motion, $\vec{C}$. Clearly this is an estimate for an adiabatic environment or where one is assumed, as the diabatic term is neglected. In addition, the local tendency term in (1) is also neglected explicitly. However, if we assume that the shape and propagation speed of an isentropic surface in a storm system remain unchanged, then by subtracting $\vec{C}$ the pressure tendency of the surface is accounted for implicitly by accelerating the flow (altering both the flow speed and direction along an isentropic surface) with respect to the cyclone center. The estimate of isentropic vertical velocity (also known as the system-relative ω) is stated as:

$$\omega_{S-R} \approx \left(\vec{V} - \vec{C}\right) \bullet \nabla_\theta p \tag{2}$$

(Saucier 1955; Moore 1993). Calculations of $\omega_{S\text{-}R}$ using this method are also included in the discussion.

b. Analyses

Calculation of the first two terms in (1) is straightforward. For the local pressure tendency term, a simple 2-h time difference centered on the time of interest is employed. The pressure transport term is then calculated at the time of interest using second-order centered finite differencing. For the diabatic term, we vertically integrate the continuity equation in isentropic coordinates:

$$\frac{\partial}{\partial t_\theta}\left(\frac{\partial p}{\partial \theta}\right) + \nabla \bullet \left(\frac{\partial p}{\partial \theta}\vec{V}\right) + \frac{\partial}{\partial \theta}\frac{\partial p}{\partial \theta}\left(\frac{d\theta}{dt}\right) = 0 \tag{3}$$

following Keyser and Johnson (1982, 1984) to arrive at:

$$\frac{\partial p}{\partial \theta}\left(\frac{d\theta}{dt}\right) = \int_\theta^{\theta_T} \nabla \bullet \left(\frac{\partial p}{\partial \theta}\vec{V}\right) d\theta + \left(\frac{\partial p}{\partial t}_{\theta_T} - \frac{\partial p}{\partial t}_{\theta}\right) \tag{4}$$

The diabatic heating term in (1) is calculated here (the left hand side of Eq. 4) by determining the vertically integrated stability flux divergence between the level of interest, θ, and an isentropic surface near the tropopause, θ_T (given by RHS term 1), and the difference between pressure tendencies at those same two levels (from RHS term 2). In the same way that the more familiar pressure coordinate form of the continuity equation can be integrated to yield a vertical motion, so too can the isentropic form (3). However, the reader should recall that, in isentropic space, this vertical motion ($\frac{d\theta}{dt}$) involves a non-conservation of θ, and thus the sought after diabatic change. Note too that, as we are basing our calculations on model output, we neglected the correction factor used by Keyser and Johnson (1982, 1984) to minimize errors in rawinsonde data and the truncation errors introduced by finite differencing of observed data.

The model output was taken from simulations with the Mesoscale Atmospheric Simulation System (MASS) version 5.10.1 (MESO 1993). This software build is a limited-area, high-resolution, hydrostatic, 21 vertical level mesoscale numerical weather prediction model formulated in x, y, σ-coordinates with the primitive equation set. Of course, to attempt a full accounting of all processes in the atmosphere, some assumptions and parameterizations become necessary; specific parameterizations included a 1 1/2 order closure scheme for the planetary boundary layer and a modified Fritsch-Chappell (1980) cumulus parameterization scheme. Details on these physics schemes can be found in MESO (1993).

Manipulation of MASS output was then achieved using the General Meteorological Package (GEMPAK; Koch et al. 1983), including a script written to calculate $\frac{d\theta}{dt}$ diagnostically during post-processing. Our focus was

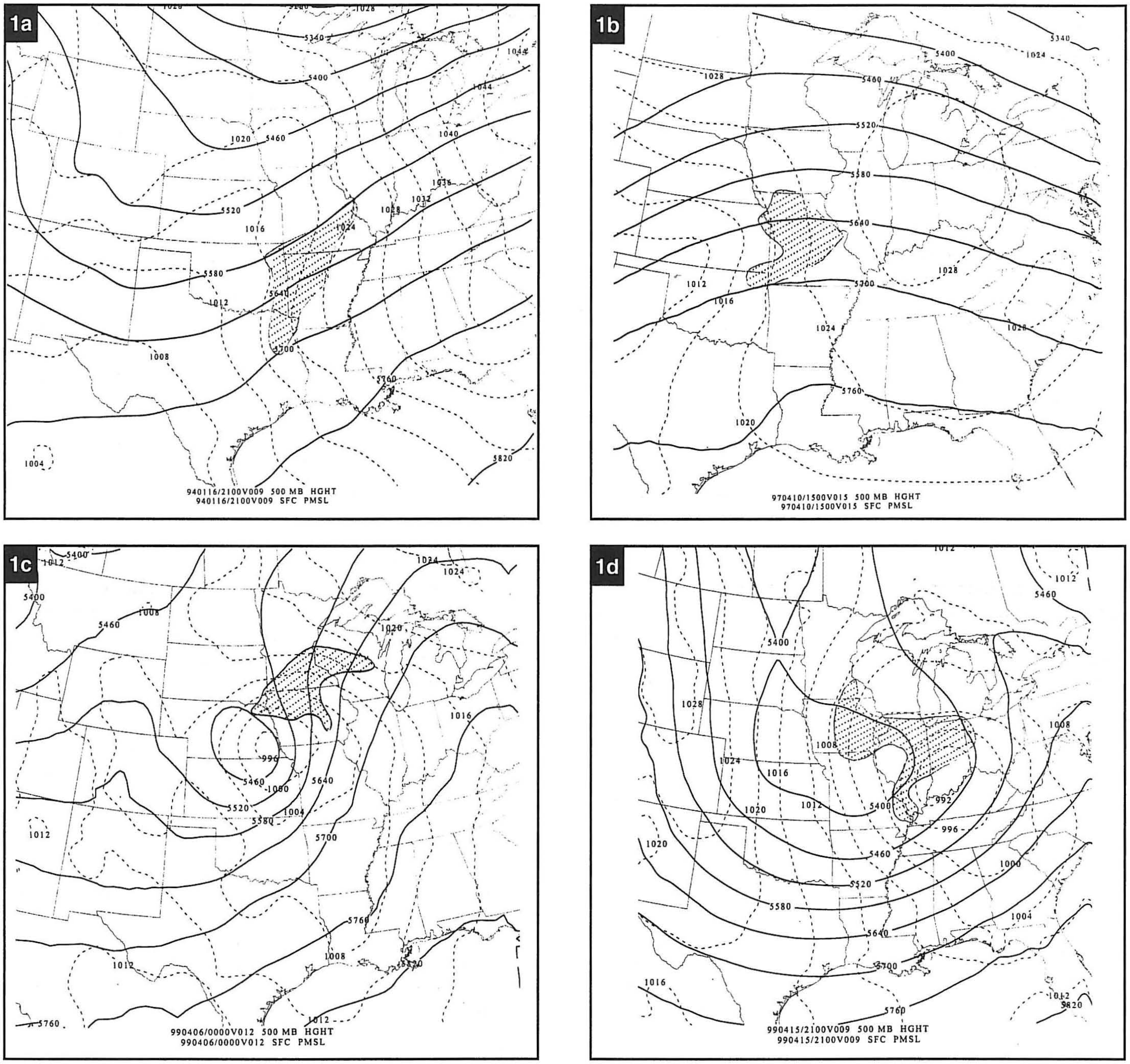

Fig. 1. MASS model output of sea level pressure (thin, dashed) contoured at 4 hPa intervals, and 500 hPa heights (bold, solid) contoured at an interval of 60 gpm, for a) 2100 UTC 16 January 1994, b) 1500 UTC 10 April 1997, c) 0000 UTC 06 April 1999, and d) 2100 UTC 15 April 1999. Stippling encloses the regions on the chosen isentropic surfaces [a) θ=296 K, b) θ=304 K, c) θ=302 K, d) θ=290 K] where the moisture criteria (relative humidity ≥ 99% and ≥ 0.5 mm of model precipitation in 1 h prior to the analysis time) were met.

those regions on lower- to mid-tropospheric isentropic levels where the relative humidity was ≥ 99% and beneath which the model had generated precipitation of ≥0.5 mm during the preceding hour. Aside from selecting output times well into the simulation (> 6 h) to ensure stable solutions, the analyzed times were chosen at random. Likewise, the selection of an isentropic surface was somewhat less arbitrary. These were determined by cross section, where we sought the isentropic level near the vertical middle of the analyzed moist region. For calculationsof $\omega_{S\text{-}R}$, $\vec{C}$ was subjectively estimated from the vorticity maximum motion on the isentropic surface of interest.

3. Results

a. Case I - 2100 UTC 16 January 1994

This was a case of a progressive wave aloft with a weak to moderate cold front and inverted trough at the surface (Fig. 1a). This system produced banded heavy

Table 1. Statistical analysis of the terms in the ω_θ equation for the 296 K surface at 2100 UTC 16 January 1994. N is the sample size, $\bar{x}$ is the sample mean, and σ is the sample standard deviation. Values for $\bar{x}$ and σ are in $\mu b\ s^{-1}$. The value for ω_θ in the $\bar{x}$ column is the summation of the three mean component terms.

	N	$\bar{x}$	σ
$\left(\frac{\partial p}{\partial t}\right)_\theta$	80	-1.95	±1.87
$\vec{V} \bullet \nabla_\theta p$	80	-2.45	±2.50
$\frac{\partial p}{\partial \theta}\frac{d\theta}{dt}$	80	-2.76	±2.91
ω_θ		-7.16	
$(\vec{V}-\vec{C}) \bullet \nabla_\theta p$	80	-3.70	±2.10

Table 2. As in Table 1, but for 304 K at 1500 UTC 10 April 1997.

	N	$\bar{x}$	σ
$\left(\frac{\partial p}{\partial t}\right)_\theta$	51	-0.53	±1.19
$\vec{V} \bullet \nabla_\theta p$	51	-1.59	±1.55
$\frac{\partial p}{\partial \theta}\frac{d\theta}{dt}$	51	-2.24	±2.49
ω_θ		-4.36	
$(\vec{V}-\vec{C}) \bullet \nabla_\theta p$	51	-3.1	±1.3

snowfall over northern Kentucky with local amounts of over 60 cm (Funk and Moore 1995). The MASS (45 km resolution) developed the system on time, but underdeveloped the precipitation area and magnitude.

For this case, the ω_θ terms were analyzed on the 296 K surface. Statistics are found in Table 1. At 9 hours into the simulation, 80 grid points on the 296 K surface exhibited the desired relative humidity and precipitation criteria noted above. Note that over this region the local tendency and diabatic terms do not offset in this case as expected by Saucier (1955) and generally assumed by researchers (e.g., Homan and Uccellini 1987) and operational forecasters alike.

While the transport term depicts rising motion, it accounts for only 34% of the total ω_θ. Including the pressure tendency term with the pressure transport term (which gives a complete *adiabatic* approximation) improves the resolution to 61% of the total ω_θ. The value

Table 3. As in Table I, but for 302 K at 0000 UTC 06 April 1999.

	N	$\bar{x}$	σ
$\left(\frac{\partial p}{\partial t}\right)_\theta$	87	-0.4	±3.5
$\vec{V} \bullet \nabla_\theta p$	87	-3.3	±3.9
$\frac{\partial p}{\partial \theta}\frac{d\theta}{dt}$	87	-2.9	±4.0
ω_θ		-6.6	
$(\vec{V}-\vec{C}) \bullet \nabla_\theta p$	87	-4.7	±5.1

Table 4. As in Table 1, but for 290 K at 2100 UTC 15 April 1999.

	N	$\bar{x}$	σ
$\left(\frac{\partial p}{\partial t}\right)_\theta$	116	-0.9	±2.4
$\vec{V} \bullet \nabla_\theta p$	116	-2.0	±3.2
$\frac{\partial p}{\partial \theta}\frac{d\theta}{dt}$	116	-0.4	±2.3
ω_θ		-3.3	
$(\vec{V}-\vec{C}) \bullet \nabla_\theta p$	116	-2.9	±4.5

of $\omega_{S\text{-}R}$ captures only 52% of the total ω_θ. In this, as with the other three cases presented, the magnitude of the standard deviation (σ) of the calculated terms approaches that of the mean. This is due largely to the mesoscale variability resolved by the model, which we have not attempted to smooth.

b. Case II - 1500 UTC 10 April 1997

Throughout this event, a slow-moving ridge aloft remained over central Missouri (Fig. 1b). A moderate surprise snowfall occurred in a band from central Missouri, through Saint Louis (10 cm) and into western Illinois. With this case, the MASS (50 km resolution) did quite well with the timing, placement, and amount of predicted snowfall. The ω_θ terms on the 304 K surface were calculated for this event from 51 grid points at 15 h into the simulation, the results of which are shown in Table 2. Again, the local tendency and diabatic terms do not offset and are both negative (ascent). Also, the transport term depicts ascent, but captures only 36% of the total ω_θ in this case. The adiabatic form employing the first two terms in (1) yields

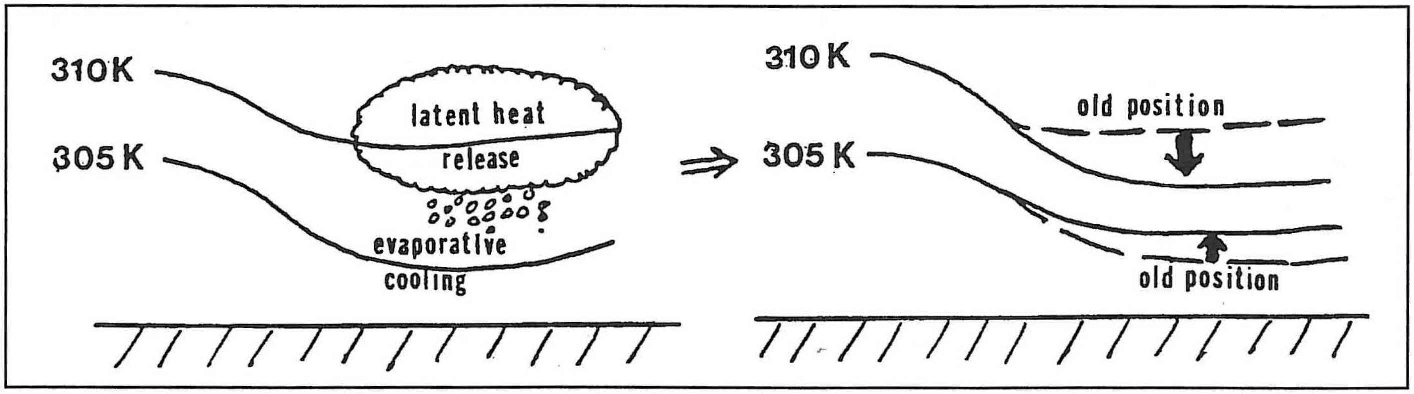

Fig. 2. Schematic diagram depicting the stability changes that can take place in the presence of diabatic heating/cooling. Reproduced from Moore (1993).

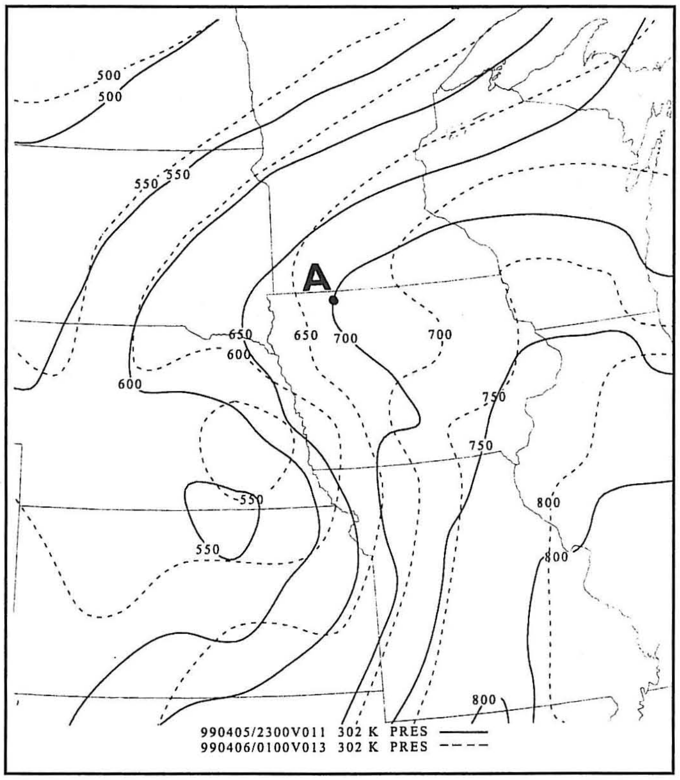

Fig. 3. Isobaric analysis on the 302 K surface for 2300 UTC 05 April 1999 (solid) and 0100 UTC 06 April 1999 (dashed). Point **A** is located within the region of preferred moisture (see Fig. 1c) and is discussed further in the text.

only 49% of the total ω_θ. The $\omega_{S\text{-}R}$ performs better here, resolving 71% of the total ω_θ.

c. Case III - 0000 UTC 06 April 1999

This event occurred with a vigorous shortwave trough aloft in association with moderate cyclogenesis over the central Plains states (Fig. 1c). The surface low was located over southeastern Nebraska and propagated northeastward with the support of a strong mid-level trough and jet streak (not shown). The MASS was run at 45 km resolution, and 87 grid points were identified at 12 h into the model run with the appropriate moisture criteria on the 302 K surface.

Even in this developing cyclonic case, the values in Table 3 demonstrate that the local pressure tendency and diabatic terms do not offset; both indicate ascent. The transport term alone accounts for 50% of the total omega, while the local pressure tendency and transport terms together account for 56% of the total omega. The system-relative ω is greater than the sum of local pressure tendency and transport terms, accounting for 71% of the total omega.

d. Case IV - 2100 UTC 15 April 1999

This is the second case associated with cyclogenesis. This system evolved over the Ohio Valley as a 994 hPa occluded low with an extensive precipitation shield by the time of this analysis (Fig. 1d). The system was moving northeastward ahead of a deep 500 hPa shortwave trough. The MASS was again run with a 45 km resolution, and 116 points were found on the 290 K surface at 9 h into the simulation that met the established moisture and precipitation criteria (Table 4).

As in the other three cases presented, the local pressure tendency and diabatic terms in (1) do not offset. However, the transport term is about 61% of the total omega, while adding the local pressure tendency and transport terms accounts for about 88% of the total omega, a fractional value identical to that of the system relative vertical velocity, $\omega_{S\text{-}R}$.

4. Summary

The terms in the isentropic vertical motion equation were evaluated using model output. Only those grid points on the chosen isentropic surface with a relative humidity of ≥ 99% and within a column with a recent (1-h) history of modeled precipitation were considered in an effort to ensure robust values for the diabatic term. A storm-relative form of ω_θ incorporating the storm motion was also computed.

Statistical analyses for four cases showed that the local pressure tendency and diabatic terms were of the same sign and *not* offsetting as is commonly assumed. This assumption springs from the concept of local diabatic heating forcing parcels to depart their former isentropic surface for higher, potentially warmer surfaces; mean-while, the former isentropic surface becomes redefined lower in the atmosphere, as the Poisson equation requires a higher pressure to accompany a higher temperature when θ is conserved (which, by definition, is the case for the isentropic *surface*). This process was described schematically (Fig. 2) by Moore (1993). Yet, this instantaneous, vertical assessment does not account for the horizontal motion of isentropic surfaces due to system translation. An example of such motion is shown in Fig. 3 with the isobar analysis on the 302 K surface at 2300 UTC 05 April 1999 and for 2 h later at 0100 UTC 06 April 1999 from Case III. At point A, the pressure decreased over the 2-h period from 700 hPa to 675 hPa as the θ surface propagated eastward. Clearly, the motion of the isentropic surface past point A gave rise to decreasing pressure (ascent) on the surface, even in the preferred moisture/precipitation area (in fact, > 2.5 mm was generated by the model near point A between 11 h and 12 h of model time), thus demonstrating the dominant influence of translation on $\frac{\partial p}{\partial t}$.

In addition, some common, simpler formulations for ω_θ were evaluated. The total mean ω_θ was of the same sign, but larger in magnitude than just the transport term by roughly a factor of two. The form of ω_θ where only the diabatic term is omitted captured about two-thirds of the total ω_θ; thus inclusion of the local pressure tendency term with the transport term gave a better result than the transport term alone. Lastly, the storm-relative version resolved nearly three-fourths of the total ω_θ in three of the four cases.

It should come as no surprise that the complete form of the ω_θ expression yields a fuller and different result than any of the single terms alone, combined two at a time, or employing some assumption on the total mean storm motion. These results serve as a reminder to use single term approximations of ω_θ with care and to demonstrate that the local pressure tendency and diabatic terms do not always offset.

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